Interference Reducing Networks

James O. Neel, *Member IEEE*, Rekha Menon, *Member IEEE*, Allen B. MacKenzie, *Member IEEE*, Jeffrey H. Reed, *Fellow IEEE*, Robert P. Gilles

Abstract— **When cognitive radios operate in a network, each link's adaptations impact the decisions of other cognitive radios which spawns an interactive decision processes. The existence of these interactive processes could potentially limit the deployment of cognitive radios as it is difficult to guarantee that the resulting behavior will avoid a tragedy of the commons, much less provide optimal performance. This paper proposes a novel design framework that ensures that cognitive radio interactions are beneficial and reduce sum network interference with each adaptation. Five different approaches to implementing algorithms that satisfy this framework are presented – two of which rely on collaboration and three which permit autonomous adaptations.**

I. Introduction

When multiple cognitive radios (CR)s operate in close proximity, the adaptation of each CR changes the state of the network, potentially influencing the decision processes of the other CRs. These interactions complicate the design and analysis of CR algorithms and could hinder the deployment of CRs. To address this problem, game theory has been proposed for the analysis and modeling of CRs [1], [2]. This paper moves beyond modeling and analysis by leveraging concepts from game theory to introduce a novel design framework for CR algorithms that ensures that these interactive processes lead to states which minimize sum network interference. We formally introduce this *interference reducing network* framework in Section 5 after covering related work in Section 2, the general application of game theory to CR networks in Section 3, and a critical game model in Section 4. Sections 6 and 7 present collaborative and greedy approaches to implementing the design scenarios, show theoretically that the algorithms satisfy the design framework, and present selected simulations of these algorithms.

J. O. Neel is with Cognitive Radio Technologies, LLC, Lynchburg VA USA (james.neel@crtwireless.com).

R. Menon is with Tyco Electronics, Lynchburg, VA USA (e-mail: rekha.menon@tycoelectronics.com).

A. B. MacKenzie is with Wireless @ Virginia Tech, Bradley Department of Electrical and Computer Engineering, Virginia Tech (e-mail: mackenab@vt.edu)

J. H. Reed is with Mobile and Portable Radio Research Group in Wireless @ Virginia Tech, Bradley Department of Electrical and Computer Engineering, Virginia Tech, and Cognitive Radio Technologies, (e-mail: reedjh@vt.edu).

R. P. Gilles is with the Department of Economics Virginia Tech, Blacksburg, VA (e-mail: rgilles@vt.edu).

II. Related Work

Past work on waveform adaptation algorithms has focused on systems with a single receiver and multiple transmitters. For the uplink of a synchronous CDMA system with a single base-station, [3] and [4] propose an algorithm where the system updates the signature sequence, s_k, of each user, k, in a round-robin fashion where each update is intended to improve the SINR of user k at the base-station which is implementing a Minimum Mean Square Error (MMSE) receiver. Specifically, given signature sequence $\mathbf{s}_k(n)$ at iteration n the updated signature sequence is given by (1)

$$\mathbf{s}_k(n+1) = \frac{\mathbf{A}_k^{-1}\mathbf{s}_k(n)}{\sqrt{\mathbf{s}_k^T(n)\mathbf{A}_k^{-2}\mathbf{s}_k(n)}} \tag{1}$$

where $\mathbf{A}_k = \sum_{j\neq k}\mathbf{s}_j\mathbf{s}_j^T + a^2\mathbf{I}_n$ where a^2 is the variance of the additive white Gaussian noise at the receiver. It is shown that the round-robin application of (1) results in a monotonically decreasing sequence of total squared correlation (TSC) values where TSC is given by (2) where $\gamma_k = \sum_{i\neq k}\sum_{j\neq k}\left(\mathbf{s}_j^T\mathbf{s}_j\right)^2$.

$$TSC = \left(\mathbf{s}_k^T\mathbf{s}_k\right)^2 + 2\mathbf{s}_k^T\left(\sum_{j\neq k}\mathbf{s}_j\mathbf{s}_j^T\right)\mathbf{s}_k + \gamma_k \tag{2}$$

Technically, this is not the same problem as we consider as there is only a single decision maker (the base station) and thus no interactive decision process. However, it is trivial to recast this problem as one where the mobiles are performing this process as other authors have done. For instance, [5] presents these same algorithms in a distributed fashion and using a general signal space approach though still with the centralized receiver. This same algorithm is applied to asynchronous CDMA systems in [6], multipath channels in [7], and multi-carrier systems in [8].

Waveform adaptation in networks with multiple collaborative receivers is investigated in [9] and [10]. Specifically, the waveforms of different mobiles communicating with different base stations are jointly controlled to minimize the total interference perceived by the mobiles. In [9], fixed points of greedy waveform adaptation algorithms in these networks are analyzed. In [10], the user's utility function is defined in terms of the weighted sum of the interference caused by the particular user at all the receivers in the system; this formulation is then used to prove the existence of Nash equilibria for the system. In each of these algorithms, all devices attempt to minimize a common function which is the sum of all interference observed in the network.

[11] analyzes waveform adaptation for a centralized network from a game theoretic perspective. It is shown that any game where the players have one of several combinations of performance metrics (such as Mean Square Error or SINR) and receiver types (such as a correlator or MSINR receiver) results in convergent NE solutions. [12] considers a dynamic frequency selection (DFS) algorithm wherein closely located radios are autonomously adapting their frequencies to minimize their perceived interference.

We can cast these papers into the operational scenarios identified in this paper as follows. Specifically, [3,5,6,7,8,11] study systems that represent specific instantiations of the *isolated cluster scenario* of Section 7. [9,10] represent special cases of the *globally altruistic scenario* of Section 6. Finally, [12] is an example of the *close proximity network scenario* of Section 7.

Beyond showing that numerous independently proposed algorithms can be bundled into a single design framework, this paper also develops new implementation approaches which greatly enhance the scalability of implementations of interference reducing network algorithms. While numerous different waveform adaptations can be imagined, for compactness simulations in this paper are restricted to examples of DFS algorithms and spreading code adaptation.

III. Game Theory and CR Networks

In a traditional game model of a CR network [2], each CR represents a player, the adaptations available to each CR form the action set of its associated player, and the CR's goal supplies the utility function for its player. A single iteration of adaptations by a network of CRs can then be modeled as a normal form game, $\Gamma=\langle N,A,\{u_i\}\rangle$, where N denotes the set of players (radios) of cardinality n and $i\in N$ specifies a particular player, A represents the adaptation space formed as $A = A_1 \times \cdots \times A_n$ where A_i specifies the action set of player i, $\{u_i\}$ is the set of utility functions, $u_i : A \to \mathbb{R}$, that describe the values which the radios assign to points in A. For notational convenience, we write a to refer to an action vector wherein each player in the game has chosen an action, a_i to refer to the action (waveform) chosen by player i, and a_{-i} to refer to the vector formed by considering all actions other than the action chosen by i.

This basic model can be extended for the purposes of further analysis by considering the specific decision rules, $d_i : A \to A_i$ that guide the radios' adaptations and the decision timings, T, at which the decisions are implemented to form the tuple, $\langle N,A,\{u_i\},\{d_i\},T\rangle$ [13]. With this model it is sometimes convenient to use $d(a)$ to refer to the collective application of $d_i(a)$ at the times specified by T. To give an intuitive feel for what we are modeling, the term "decision rule" refers to some well-defined process that controls a CR's adaptations which has presumably been designed so as to increase the value of u_i with each adaptation. For example, a decision rule may specify discrete steps up or down in response to observed channel conditions or may specify a sequence of alternate frequencies to try when interference is detected. However, some CRs are not implemented with well-defined decision rules and are instead only lightly governed by goals, policies, and available adaptations. To handle both of these cases, we restrict our design framework to a set of decision rules which we term *autonomously rational* which satisfy (3).

$$b_i \in d_i(a), b_i \neq a_i \Rightarrow u_i(b_i, a_{-i}) > u_i(a_i, a_{-i}) \tag{3}$$

A game theorist would refer to the behavior which results from the use decision rules of this form as a *better response dynamic*. Similarly, an *exhaustive better response dynamic* occurs when all decision rules satisfy (4).

$$a_i \notin d_i(a) \text{ if } \exists b_i \in A_i : u_i(b_i, a_{-i}) > u_i(a_i, a_{-i}) \tag{4}$$

A. Steady State Analysis of CR Networks

We leverage techniques from game theory to characterize the steady-states and the convergence of the CR networks designed using the proposed framework. In game theory, the traditional "steady-state" concept is the *Nash equilibrium* (NE) which is an action vector a^* where $u_i(a^*) \geq u_i(b_i, a_{-i}^*)$ $\forall i \in N, b_i \in A_i$. It is clear that the NE of the normal form game model of a CR network are fixed points for autonomously rational decision rules and are the *only* fixed points for exhaustive better response dynamics.

<u>Theorem 1</u>: Given CR network $\langle N,A,\{u_i\},\{d_i\},T\rangle$ where all players are autonomously rational, if a^* is an NE for $\langle N,A,\{u_i\}\rangle$, then a^* is a fixed point for d.

Proof: Suppose a^* is not a fixed point. Then for some $i\in N$, there must be some $b_i \in d_i(a^*)$ with $b_i \neq a_i^*$ such that $u_i(b_i, a_{-i}^*) > u_i(a_i^*, a_{-i}^*)$. But this contradicts the assumption that a^* is an NE. Therefore, a^* must be a fixed point for d.■

<u>Theorem 2</u>: Given CR network $\langle N,A,\{u_i\},\{d_i\},T\rangle$ where all decision rules are exhaustive better responses, a^* is a fixed point of d iff a^* is an NE for $\langle N,A,\{u_i\}\rangle$.

Proof: Sufficiency is supplied by Theorem 1. Suppose a^* is a fixed point of d but not an NE. As a^* is not an NE, there is some player i with $a_i^{**} \neq a_i^*$ such that $u_i(a_i^{**}, a_{-i}^*) > u_i(a_i^*, a_{-i}^*)$, but this contradicts the assumption of an exhaustive better response.■

B. Convergence Analysis of CR Networks

To examine convergence properties, we consider the concept of improvement paths in games and the *Finite Improvement Property* (*FIP*) as defined in [14]. A *path* in Γ is a sequence $\gamma = (a^0, a^1, \ldots)$ such that for every $k \geq 1$ there exists a unique player such that the strategy combinations (a^{k-1}, a^k) differ in exactly one coordinate. An *improvement path* is a path such that for all $k\geq1$, $u_i(a^k) > u_i(a^{k-1})$ where i is the unique deviator at step k. The improvement path, $\gamma = (a^0, a^1, \ldots, a^k)$, is *exhaustive* if there is no a^{k+1} such that an improvement path exists from a^k to a^{k+1}. A game, $\Gamma=\langle N,A,\{u_i\}\rangle$, is said to have to have the *finite improvement property* (FIP) if all improvement paths in Γ are finite.

An example of a game with FIP is shown in matrix form representation in Figure 1. All of the possible action vectors in the game's action space are arrayed in a matrix such that player 1's actions (the first component of the action vector) are given by the rows of the matrix and player 2's actions (the second component of the action vector) are given by the columns of the matrix. Each cell in this matrix is thus determined by a unique action vector (row, column) and is filled with the payoff vector associated with the cell's action vector. A complete listing of the improvement paths for this game is given in Table 1. From our exhaustive listing, we can readily establish that this game has FIP and that the longest path has a length of 3.

	A	B
a	(5,5)	(-1,10)
b	(10,-1)	(0,0)

Figure 1: Prisoners' Dilemma Game Matrix for Improvement Path Analysis.

Table 1: Improvement Paths for Game Presented in Figure 1.

$\gamma_1 = ((a, A), (a, B))$	$\gamma_3 = ((b, A), (b, B))$
$\gamma_2 = ((a, A), (b, A))$	$\gamma_4 = ((a, B), (b, B))$
$\gamma_5 = ((a, A), (a, B), (b, B))$	
$\gamma_6 = ((a, A), (b, A), (b, B))$	

However, not all games have FIP as illustrated in Figure 2 where an infinite improvement path is given by the sequence $\left((a,A),(b,A),(b,B),(a,A),\ldots\right)$.

	A	B
a	(0,1)	(1,0)
b	(1,0)	(0,1)

Figure 2: A Game without FIP

Showing that a game has FIP permits several immediate insights such as shown in Theorems 3 and 4 which are well-known in the game theory community.

Theorem 3: All games with FIP have at least one NE.
Proof: By FIP there must be at least one action tuple, a^*, from which there exists no profitable unilateral deviation. This action tuple a^* must be an NE as there exists no other $a \in A$ such that $u_i\left(a_i, a_{-i}^*\right) > u_i\left(a^*\right)$.■

Theorem 4: All exhaustive improvement paths in a game with FIP end in an NE.
Proof: Suppose the improvement path does not terminate, then the sequence is an infinite improvement path in contradiction of the assumption of FIP. Suppose an exhaustive improvement path terminates in action tuple, a^* which is not an NE. Then this contradicts the assumption that the sequence is exhaustive as for some player i there exists an a_i such that $u_i\left(a_i, a_{-i}^*\right) > u_i\left(a_i^*, a_{-i}^*\right)$.■

Thus for systems which can be modeled as having exhaustive better response decision rules and restrict adaptations to one radio at a time, showing that the network's game model has FIP is sufficient to be assured of convergence to an NE. However, this result can be strengthened to a more general timing model – the *asynchronous timing model* where for each $t \in T$ each player, i, has probability $0<p_i<1$ of implementing its decision process (perhaps due to an internal random timer). Let M_Γ be the Markov chain that results when players in $\Gamma=\langle N,A,\{u_i\}\rangle$ operate under an asynchronous timing model and apply exhaustive better response decision rules to the previous state (action tuple) to form the next state. Without knowing the radios' specific decision rules, we cannot write a transition matrix for M_Γ. However, we can make some inferences about M_Γ.

Theorem 5: An action tuple, a^*, is an absorbing state for M_Γ iff a^* is an NE.
Proof: Suppose a^* is an NE, but not an absorbing state. Then there must exist some other state to which there is a nonzero probability of the system transitioning from a^*. Under the assumption of better response decision rules in M_Γ, nonabsorption implies that there is some player i such that $a_i \neq a_i^*$, $u_i\left(a_i, a_{-i}^*\right) > u_i\left(a_i^*, a_{-i}^*\right)$. However this contradicts the assumption that a^* is an NE. Now suppose a^* is an absorbing state but not an NE. As a^* is not an NE, there is some player i with $a_i \neq a_i^*$ such that $u_i\left(a_i, a_{-i}^*\right) > u_i\left(a_i^*, a_{-i}^*\right)$. As M_Γ assumed $p_i > 0 \forall i \in N$ and exhaustive better responses, the probability of transitioning from a^* must be greater than 0. However, this contradicts the assumption that a^* is an absorbing state.■

Theorem 6: If $\Gamma=\langle N,A,\{u_i\}\rangle$ has FIP, then M_Γ is an absorbing Markov chain.
Proof: A Markov chain is an absorbing Markov chain if from every state there exists a sequence of state transitions of nonzero probability that terminates in an absorbing state. Given $a \in A$, let γ_a be an exhaustive improvement path that terminates in an NE, $n(\gamma_a, t)$ be the player that adapts at step t of γ_a, and $n(\gamma_a)$ be the sequence of adapting players in γ_a. Under an asynchronous timing model, the probability that only $n(\gamma_a, t)$ adapts at a particular iteration is given by $p_{n(\gamma_a,t)} \prod_{k \in N \setminus n(\gamma_a,t)} (1-p_k)$ and that starting from a, the sequence of adapting players is $n(\gamma_a)$ is given by $\prod_{n(\gamma_a,t) \in n(\gamma_a)} \left[p_{n(\gamma_a,t)} \prod_{k \in N \setminus n(\gamma_a,t)} (1-p_k) \right]$. As this last expression is greater than zero and as every NE in Γ is an absorbing state of M_Γ, γ_a specifies a sequence of state transitions of nonzero probability that terminates in an absorbing state. By Theorem 4 there exists an γ_a for every $a \in A$, thus from every state there exists a sequence of state transitions of nonzero probability that terminates in an absorbing state.■

IV. Potential Games

While the knowledge that the game model of a CR network has FIP permits valuable insights into what kinds of decision rules will converge to NE (all exhaustive better responses under unilateral or asynchronous timings), showing that the game model has FIP can be arduous. Fortunately, a readily identifiable class of games known as *potential games* can be shown to have FIP. A normal form game, $\Gamma=\langle N,A,\{u_i\}\rangle$, is said to be an *exact potential game* if there exists a function, $V:A\rightarrow\mathbb{R}$, known as an *exact potential function*, that satisfies $u_i(b_i,a_{-i})-u_i(a_i,a_{-i})=V(b_i,a_{-i})-V(a_i,a_{-i})\ i\in N,\forall a\in A$. An obvious way to construct an exact potential game is for every player to have the same utility function, e.g., as shown in (5) where $C:A\rightarrow\mathbb{R}$.

$$u_i(a)=C(a) \tag{5}$$

Such a game is called a *coordination game* and has and exact potential function given by $V = C$. Another exact potential game form was introduced in [15] where every player's utility function is of the form shown in (6)

$$u_i(a)=\sum_{j\in N\setminus i} w_{ij}(a_i,a_j)-S_i(a_i) \tag{6}$$

where $w_{ij}:A_i\times A_j\rightarrow\mathbb{R}$ and $S_i:A_i\rightarrow\mathbb{R}$ such that for every $(a_i,a_j)\in A_i\times A_j$, $w_{ij}(a_i,a_j)=w_{ji}(a_j,a_i)$. Such a game is called a *bilateral symmetric interaction* (BSI) game and has an exact potential function given by (7).

$$V(a)=\sum_{i\in N}\sum_{j=1}^{i-1} w_{ij}(a_i,a_j)-\sum_{i\in N}S_i(a_i) \tag{7}$$

(7) can be verified as an exact potential function for BSI games by evaluating the change in (6) and (7) when an arbitrary player i changes its action from a_i to b_i while the vector of the remaining players actions, a_{-i} is held constant.

$$\begin{aligned}u_i(a_i,a_{-i})-u_i(b_i,a_{-i}) &= \sum_{j\in N\setminus i} w_{ij}(a_i,a_j)-\sum_{j\in N\setminus i} w_{ij}(b_i,a_j)\\ &\quad -S_i(a_i)+S_i(b_i)\\ V(a_i,a_{-i})-V(b_i,a_{-i}) &= \sum_{j\in N\setminus i} w_{ij}(a_i,a_j)-\sum_{j\in N\setminus i} w_{ij}(b_i,a_j)\\ &\quad -S_i(a_i)+S_i(b_i)\end{aligned}$$

Additional potential game forms are given in Chapter 5 of [13].

Theorem 7: If $\Gamma=\langle N,A,\{u_i\}\rangle$ is an exact potential game with potential function V and finite A, then Γ has FIP.

Proof: (Along the lines of a proof given in [14]) Suppose $\Gamma=\langle N,A,\{u_i\}\rangle$ is an exact potential game with potential V. Now consider any improvement path $\gamma=(a^0, a^1,\ldots)$ in A. Then $u_i(a^{k+1})>u_i(a^k)$ where i is the unique deviator at step k+1. As Γ is an exact potential game, $u_i(a^{k+1})>u_i(a^k)\Rightarrow V(a^{k+1})>V(a^k)$. Then $V(a^0)< V(a^1)< \cdots$ and $V(\gamma)$ forms a monotonically increasing sequence. Since A is finite and $V(\gamma)$ is monotonic, γ must be finite.■

While potential games with infinite action spaces do not generally possess the FIP, autonomously rational decision rules under unilateral timing still yield a monotonically increasing V. If these infinite action spaces are bounded, some decision rules can still be shown to converge by applying Convergence Theorem A from [16].

Theorem 8: Let $d{:}A\rightarrow A$ determine an algorithm that given a point a^0 generates a sequence $(a^k)_0^\infty$ through the iteration $a^{k+1}\in d(a^k)$. Let a solution set, $S^*\subset A$, be given. Suppose (1) All points $(a^k)_0^\infty$ are in a compact set $S\subset A$. (2) There is a *continuous function* $\alpha:A\rightarrow\mathbb{R}$ such that: (a) if $a\notin S^*$, then $\alpha(a')>\alpha(a)\forall a'\in d(a)$; (b) if $a\in S^*$, then $\alpha(a')\geq\alpha(a)\ \forall a'\in d(a)$. (3) d is closed at a if $a\notin S^*$. Then either the recursion $a^{k+1}\in d(a^k)$ arrives at a solution (fixed point), or the limit of any convergent subsequence of $(a^k)_0^\infty$ is in S^*.

Proof: A proof of this theorem is given in [16].■

Theorem 9: Given an exact potential game with a compact action space A and potential V, if the following conditions hold, then the recursion $a^{k+1}\in d(a^k)$ converges.

1) V is continuous; 2) d_i is autonomously rational $\forall i\in N$; 3) d is closed.

Proof: This is just an application of Theorem 8 with $\alpha=V$. Thus d converges to some $a^*\in D^*$ where D^* are the set of fixed points for d.■

Finally, potential games also permit the following well-known characterization of a game's NE.

Theorem 10: Given exact potential game, $\Gamma=\langle N,A,\{u_i\}\rangle$ with potential function V, if $a^*=\max_{a\in A}V(a)$ then a^* is an NE.

Proof: Suppose $a^*=\max_{a\in A}V(a)$ is not an NE. As a^* is not an NE, there is some player i with $a_i^{**}\neq a_i^*$ such that $u_i(a_i^{**})>u_i(a_i^*)$. But this implies that $V(a_i^{**})>V(a_i^*)$ and contradicts the assumption that $a^*=\max_{a\in A}V(a)$. Therefore a^* must be an NE.■

Note that the set of global maximizers of a game's potential function need not capture all of the NE as local maximizers of V can also be NE and "elusive NE" have been shown to exist in potential games with infinite action spaces [17]. A lengthier discussion of the properties of potential games is given in Chapter 5 of [13].

V. Interference Reducing Networks

Unfortunately, simply because decision rules converge to a maximizer of the potential function, this does not imply that the associated NE is desirable. For example the game previously shown in Figure 1 is an exact potential game with a potential function as shown in Figure 3. Note that (b, B) maximizes V and is the game's only NE, but (a, A) is preferable to (b, B) for both players.

$V(\cdot)$	A	B
a	0	5
b	5	6

Figure 3: Exact Potential Function for Game in Figure 1.

Thus while applying potential games to the design of CR networks will ensure convergence to a set of readily defined equilibria for a broad range of decision rules, there is no guarantee that these equilibria are desirable. This section addresses this problem via a novel framework that ensures the game model of a CR network has a potential function whose maximizers minimize sum network interference.

Let $I_i(a), I_i : A \to \mathbb{R}$, represent the interference that CR i observes based on the actions of the other radios in the network. From these observations, we form the *network interference function*, $\Phi(a)$, by summing these terms as shown in (8).

$$\Phi(a) = \sum_{i \in N} I_i(a) \tag{8}$$

A CR network, $\langle N, A, \{u_i\}, \{d_i\}, T \rangle$, is said to be an *interference reducing network* (IRN) if all unilateral autonomously rational adaptations decrease the value of Φ. From our discussion in Section 4, it is apparent that an IRN can be implemented via potential game networks when $\Phi \propto -V$. In the following sections we examine how the goals and observation processes of CRs can be shaped to achieve this result.

VI. Altruistic Interference Reducing Networks

An obvious technique to achieve an IRN is to assign each radio coordination game goals of the form $u_i(a) = -\Phi(a)$. We term such an implementation a *globally altruistic IRN* as each radio is directly working to reduce the sum of every radio's observed interference. While this is a potential game of the desired form, i.e., $V(a) = -\Phi(a)$, an implementation of such a CR network is not very practical as each radio must distribute to every other radio its observed interference $I_i(a)$. While this approach is likely unsuitable for implementation due to scalability concerns, [18] defines one possible means for distributing the global interference measurements throughout the network, namely a radio environment map to which each radio can poll and report observations. Further, several proposed algorithms effectively realize a globally altruistic IRN. [19] proposes a DFS algorithm where the radios are guided by minimization of the sum network interference but with estimations of other radios' interference levels. Likewise, the algorithms of [9] also satisfy the conditions of a globally altruistic network as applied to spreading code adaptations.

Limiting the scalability issues, an altruistic IRN can also be created if the radios consider the interference levels observed by subsets of radios. Let $\Im_i \subseteq N \setminus i$ denote the set of radios where the signal level of radio i is strong enough to produce non-negligible interference. Then if each radio is guided by goals of the form shown in (9)

$$u_i(a) = -\sum_{k \in \Im_i} I_k(a) - I_i(a) \tag{9}$$

an exact potential function is still given by $V(a) = -\Phi(a)$ as $\sum_{k \in N \setminus \Im_i} I_k(a) = 0$ in our model and

$$u_i(a_i, a_{-i}) - u_i(b_i, a_{-i}) = \sum_{k \in \Im_i} I_k(b_i, a_{-i}) - \sum_{k \in \Im_i} I_k(a_i, a_{-i}) - I_i(a_i, a_{-i}) + I_i(b_i, a_{-i})$$

$$V(a_i, a_{-i}) - V(b_i, a_{-i}) = \sum_{k \in \Im_i} I_k(b_i, a_{-i}) - \sum_{k \in \Im_i} I_k(a_i, a_{-i}) - I_i(a_i, a_{-i}) + I_i(b_i, a_{-i})$$

In general, this approach will consume less bandwidth to distribute the interference information, but in dense networks, the overhead can still be quite significant.

VII. Greedy Interference Reducing Networks

Unfortunately, altruistic IRNs necessarily incur significant information distribution costs and greedy algorithms, while incurring less overhead, cannot generally be assured of having desirable steady-states. However, by restricting our networks to a certain set of conditions, which we term *bilateral symmetric interference* (BSI), we can create greedy algorithms which realize IRNs. Thus when BSI holds, we can get the benefit of the coordination equilibria, but without the overhead by implementing greedy algorithms!

We say that two CRs, $j, k \in N$, exhibit BSI if $g_{jk} p_j \rho(a_j, a_k) = g_{kj} p_k \rho(a_k, a_j)$ $\forall a_j \in A_j, \forall a_k \in A_k$ where p_k is the transmission power of radio k's waveform, g_{kj} is the link gain from the transmission source of radio k's signal to the point where radio j measures its interference, $\rho(a_k, a_j)$ is the fraction of radio k's signal power that interferers with radio j. In general, $\rho(a_k, a_j)$ is determined by the absolute value of the correlation between the signal space basis functions modulated by a_k and a_j.

Now assume that the radios are guided by a greedy goal as shown in (10).

$$u_i(a) = -I_i(a) = \sum_{j \in N \setminus i} g_{ji} p_j \rho(a_j, a_i) \tag{10}$$

Under the assumption of BSI (10) is an example of a goal for a bilateral symmetric interaction game as shown in (6) where $w_{ij}(a_i, a_j) = g_{ji} p_j \rho(a_j, a_i)$. Thus, an exact potential function exists for these games of the form shown in (11) and $\Phi(a) = -2V(a)$.

$$V(a) = -\sum_{i \in N} \sum_{j=1}^{i-1} g_{ji} p_i \rho(a_j, a_i) \tag{11}$$

Thus sequences of unilateral greedy adaptations will increase V and decrease Φ which means that when BSI holds, algorithms that seek to minimize a radio's own interference will implement an IRN under unilateral and asynchronous timings and will converge to a minimizer of Φ for exhaustive greedy better responses.

To establish that the BSI condition holds we must show that $\rho(a_k,a_j) = \rho(a_j,a_k)$ and $g_{kj}p_j = g_{jk}p_k$. We frequently encounter situations where $\rho(a_k,a_j) = \rho(a_j,a_k)$ such as adjacent and co-channel interference from channel (frequency) selection or the cross correlation between signature sequences. However there are some waveform adaptations for which this equality does not hold, most notably in beam forming applications. Additionally, differences in power or link gains can lead to violations of the BSI condition. Nonetheless, the following considers three different scenarios where BSI holds.

A. *Isolated Clusters with Power Control*

Encountered in infrastructure based networks employing code or frequency reuse, in the isolated clusters scenario, the system consists of a set of clusters C for which the following operational assumptions hold:

1. Perhaps through judicious frequency or code reuse between clusters, each radio i is operating in a cluster $c \in C$ for which $\Im_i$ is a subset of c.
2. The cluster head (or access point or base station) enforces a uniform receive power, r_c, on all radios k for signals transmitted to the cluster head.
3. Waveforms are restricted to those waveforms for which $\rho(a_k,a_j) = \rho(a_j,a_k)$.
4. The radios implement autonomously rational decision rules guided by (10) as measured at the cluster head with either unilateral or asynchronous timing.

These assumptions are sufficient to establish that BSI holds. By assumption 1, $g_{jk}p_j\rho(a_j, a_k) = g_{kj}p_k\rho(a_k, a_j) = 0$ whenever j and k are in different clusters. For radios in the same cluster, assumption 2 implies $g_{kj}p_j = g_{jk}p_k$ and assumption 3 requires $\rho(a_k,a_j) = \rho(a_j,a_k)$. Thus BSI holds for all pairs of radios within a cluster and across clusters, and assumption 4 supplies the requisite requirements of autonomous rationality and timing.

Though formulated differently and generally considering specific decision rules instead of any greedy decision rule guided by (10), such a set of assumptions is implicitly utilized in [3, 5-8, 11] for spreading code adaptations. To give an intuitive feel for how such a system performs Figure 4 depicts the results of a simulation of seven radios adapting their spreading codes over the surface of a six-dimensional hypersphere code-adapting (so orthogonal codes are not achievable) guided by (10) as measured at a common cluster head with a constant received power of -50 dBm with one radio adapting at a time and starting from an initial random assignment of spreading codes. The top plot shows the measured interference levels for the each of the cognitive radio and the bottom plot shows $\Phi(a)$ for the network. Note that each adaptation reduces the value of $\Phi(a)$ as predicted by virtue of being an IRN although individual radios interference levels may increase.

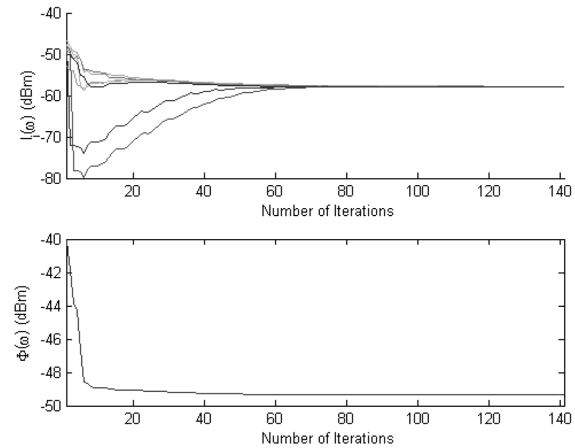

Figure 4: Simulation of seven code-adapting CRs in an isolated cluster.

B. *Close Proximity Networks*

In this operational scenario it is assumed that the radios are operating as an ad-hoc network in sufficiently close proximity and transmitting with sufficiently similar power levels that waveform correlation dominates making the distance and transmitted power effects negligible. Such a scenario may arise in a network of closely spaced WLAN devices where the presence of any in-band energy triggers a collision event, although such a network would constitute an *effective* IRN as opposed to the strict IRN. Under these assumptions differences in received powers are negligible and (10) is equivalent to (12) .

$$u_i = -\sum_{j \in N \setminus i} \rho\left(a_j, a_i\right) \tag{12}$$

If we assume that $\rho(a_k,a_j) = \rho(a_j,a_k)$, the system satisfies the BSI condition and forms an IRN for autonomously rational decision rules for unilateral or asynchronous timing.

C. *Controlled Observation Processes*

It is also possible to achieve the BSI condition by controlling the observation processes of the CRs in addition to the goals and decision processes. For instance suppose a network consists of a collection of 802.11 clusters where each cluster is controlled by a fixed access node whose channel selections are guided by (10). To this we add the restriction that the only observation made by an access node are the received signal power of the RTS/CTS signals transmitted by other access nodes as detected by the cognitive access node. By making this restriction of observations, we know the following:

- All observed signals are transmitted at the same power level, p, as RTS/CTS messages are generally transmitted at maximum power to clear out hidden nodes.
- A symmetric link gain exists between the transmission points and observation points of pairs of CRs. Specifically, the gain from access node j to access node k, g_{jk}, is the same as the gain from access node k to access node j, g_{kj}.[1]

[1] It is permissible that link gains between access nodes are frequency selective, but frequency selectivity of the gains must be reciprocal as well,

Then by restricting the adaptation choices to a selection of channels, we have

$$\rho\left(a_j, a_k\right) = \begin{cases} 1 & a_j = a_k \\ 0 & a_j \neq a_k \end{cases} \tag{13}$$

for orthogonal channel sets (such as in the UNII bands) and

$$\rho\left(a_j, a_k\right) = \max\left\{\left(B - \left|a_j - a_k\right|\right)/B, 0\right\} \tag{14}$$

for non-orthogonal channel sets (such as in the ISM bands) where B is the signal bandwidth. Such a network could be encountered in an enterprise WLAN installation where multiple access nodes with the same maximum transmit power level are deployed throughout a building or in an infrastructure based WRAN deployment, thereby ensuring that the access nodes all have the same maximum transmit power.

To illustrate the operation of such a system, Figures 5 and 6 depict the results of two simulations of thirty access nodes randomly distributed over 1 km^2 with a path loss exponent of 3 guided by (10) and unilaterally adapt their clusters' center frequencies over 10 MHz of nonorthogonal channels while supporting 1 MHz bandwidth signals implying correlations of the form shown in (14). The top plot depicts the operating frequencies of each radio starting from random initial distributions of frequencies; the middle plot shows the evaluation of the goals of all the radios in the network; the bottom plot shows the value of Φ. While the networks converge to different steady-state frequency distributions, in both cases, $\Phi(\omega)$ forms a monotonically decreasing sequence as predicted by the IRN framework. The results of a simulation with the same access points but with asynchronous timing and the European UNII channel set (implying correlations take the form shown in (13) is shown in Figure 7. While Φ still trends down, it no longer does so monotonically as pairs of radios will occasionally adapt onto the same channel raising net interference. Nonetheless, because this system forms an absorbing Markov chain as predicted in Section 5, it eventually converges to a frequency vector that is a minimizer of Φ.

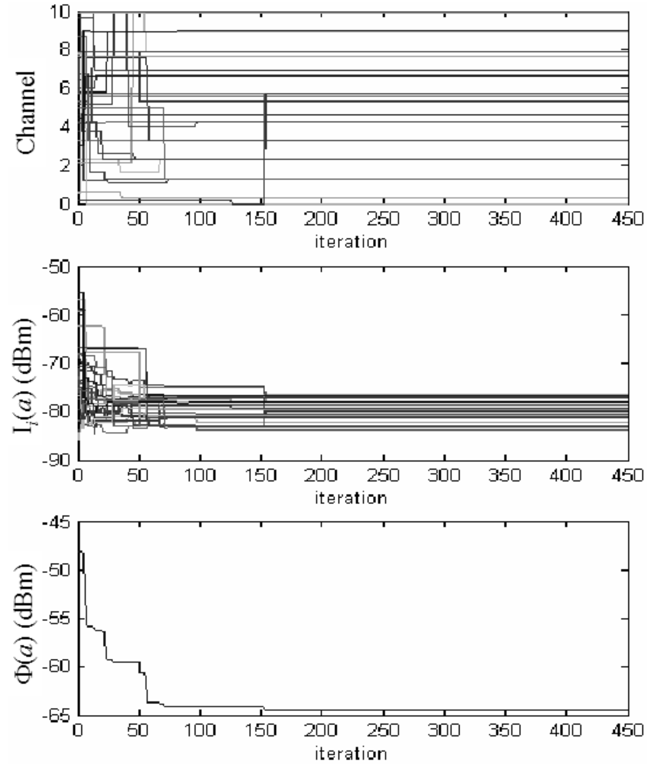

Figure 5: 30 randomly distributed DFS nodes adapting to interference measured at the transmitter.

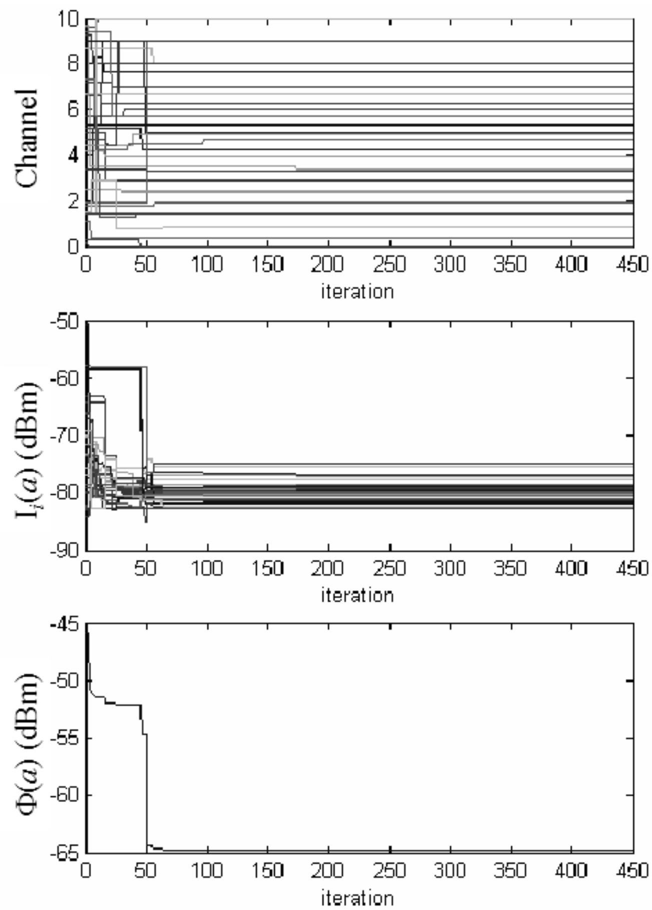

Figure 6: Simulation of system in Figure 5 with different initial frequencies.

i.e., $g_{ik}\left(f_i\right) = g_{ki}\left(f_i\right)$. For purposes of analysis, this frequency selectivity can be considered a part of $\rho(a_k,a_j)$.

Table 2: Scenario Comparisons

Scenario	Special Topology	Waveform Restrictions	Observation Restrictions	Relative Overhead
Globally Altruistic	N	N	N	Very High
Locally Altruistic	N	N	N	High
Isolated Cluster	Y	Y	N	Low
Close Proximity	Y	Y	N	Low
Controlled Observation	N	Y	Y	Very Low

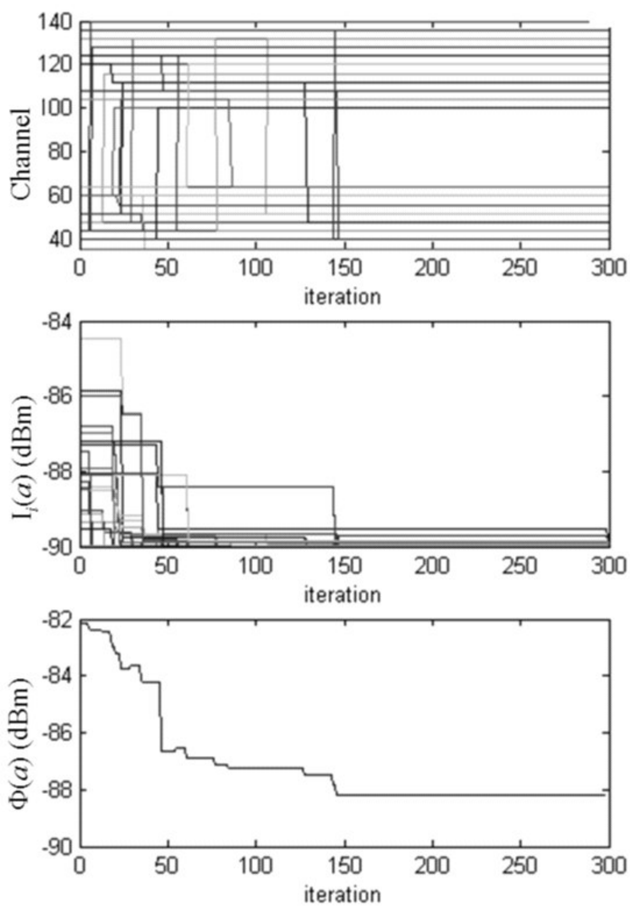

Figure 7: Controlled Observation Scenario in UNII bands under Asynchronous Timing.

VIII. Summary and Directions for Future Research

This paper proposed a powerful new framework for the design of cognitive radio algorithms – the interference reducing network – for which each adaptation improves network performance for all autonomously rational decision rules under the five scenarios considered in Sections 6 and 7. In the altruistic scenarios the radios attempt to minimize other radios' observed interference in addition to their own. By doing this, no special requirements need to be placed on the topology of the network, the available waveforms nor the observation processes of the radios. However, the network must consume significant bandwidth to distribute the radios' observations. In the greedy scenarios, the radios act only to reduce their own interference. This eliminates the need to distribute observations other than between reception and decision points of a single link. However, to realize an IRN in the greedy scenarios the BSI condition must hold which places restrictions on the topology, waveforms, and/or observation processes of the radios. These tradeoffs between these approaches to implementing an IRN are summarized in Table 2. Noting the fundamental tradeoff between external and internal observations, namely complexity versus generalizability, an interesting line of research becomes immediately apparent – how can cognitive radios recognize when they are operating under the BSI condition so higher efficiency networks can be implemented?

This is not an exhaustive listing of IRN implementation scenarios. For instance, receive beam forming adaptations would violate the reciprocal correlation property, yet will implement an IRN for any network topology and greedy algorithms.[2] The examples considered in this paper should represent only a fraction of CR networks that could be designed to achieve an IRN and only a fraction that satisfy the BSI condition. It should be possible to identify additional IRNs by considering alternate topology and observation constraints, adaptations beyond frequency and spreading codes, and combinations of constraints and multiple adaptable waveform parameters.

Acknowledgment

This material is based in part on work sponsored by basic research grant no. N000140310629 from the Office of Naval Research, the IREAN program at Virginia Tech, NSF CAREER grant no. CNS-0448131, and the support of MPRG and Wireless @ Virginia Tech industrial affiliates.

References

[1] J. Neel, R. Buehrer, J. Reed, and R. Gilles, "Game Theoretic Analysis of a Network of Cognitive Radios," *Midwest Symposium on Circuits and Systems 2002*.

[2] J. Neel, J. Reed, R. Gilles, "Game Models for Cognitive Radio Analysis," *SDR Forum 2004 Technical Conference*, Nov. 2004.

[3] S. Ulukus and R.D. Yates, "Iterative construction of optimum signature sequence sets in synchronous CDMA systems," *IEEE Transactions on Information Theory*, vol. 47, no. 5, July 2001, pp. 1989-1998.

[4] C. Rose, S Ulukus, R. Yates, "Interference avoidance for wireless systems," *Spring Vehicular Technology Conference Proceedings 2000*, pp. 901-906, 2000.

[2] Consider any network topology implementing receive beam forming. Each self-interested adaptation guided by reducing received interference will presumably decrease the interference measured at the adapting device while having no impact on the other cognitive radios in the network. Thus, given a sequence of such adaptations, the sum of all measured interference levels, $\Phi(a)$, forms a monotonically decreasing sequence.

[5] C. Rose, S. Ulukus, and R. D. Yates, “Wireless systems and interference avoidance,” *IEEE Transactions on Wireless Communications*, vol. 1, no. 3, July 2002, pp. 415-427.

[6] S. Ulukus, R. D. Yates, “Signature Sequence Optimization in Asynchronous CDMA Systems,” *Proc. IEEE Int. Conf. Communications*, vol. 2, June 2001, pp. 545-549.

[7] J. I. Concha, S. Ulukus, “Optimization of CDMA Signature Sequences in Multipath Channels,” *Proc. IEEE Vehic. Tech. Conf.* vol. 3, May 2001, pp.1978-1982.

[8] D. Popescu, C. Rose, “Interference Avoidance Applied to Multiaccess Dispersive Channels,” *Proc. Conf. Int. Symp. Info. Theory,* July 2002.

[9] O. Popescu, C. Rose, “Sum Capacity and TSC Bounds in Collaborative Multi-Base Wireless Systems”, *IEEE Transactions on Information Theory*, vol. 50, no. 10, pp. 2433-2438, Oct. 2004.

[10] C. Sung, K. Leung , “On the stability of distributed sequence adaptation for cellular asynchronous DS-CDMA systems,” *IEEE Transactions on Information Theory*, vol. 49, no. 7, July 2003, pp. 1828-1831.

[11] J. Hicks, A. MacKenzie, J. Neel, J. Reed, “A Game Theory Perspective on Interference Avoidance,” *IEEE Global Telecommunications Conference*, vol.1, Dec. 2004, pp. 257-261.

[12] J. Neel, R. Menon, A. MacKenzie, J. Reed, "Using Game Theory to Aid the Design of Physical Layer Cognitive Radio Algorithms," *Conference on Economics, Technology and Policy of Unlicensed Spectrum*, May 16-17 2005.

[13] J. Neel, “Analysis and Design of Cognitive Radio and Distributed Radio Resource Management Algorithms,” PhD Dissertation, Virginia Tech, Sep. 2006.

[14] D. Monderer, and L. Shapley, "Potential Games," *Games and Economic Behavior* **14**, pp. 124-143, 1996.

[15] T. Ui., “A Shapley Value Representation of Potential Games,” *Games and Economic Behavior*, **31**, 121-135, 2000.

[16] W. Zangwill, **Nonlinear Programming: A Unified Approach**, Prentice Hall, Englewood Cliffs NJ, 1969.

[17] J. Hicks, A. B. MacKenzie, “A Convergence Result for Potential Games,” *11th Int’l Symp. on Dynamic Games and Applications*, Dec. 18-21, 2004.

[18] Y. Zhao, B. Le, J. Reed, “Network Support – The Radio Environment Map,” **Cognitive Radio Technology**, B. Fette, ed., Elsevier, August 11, 2006.

[19] N. Nie, C. Comaniciu, “Adaptive channel allocation spectrum etiquette for cognitive radio networks,” *DySPAN2005*, Nov. 2005 pp. 269-278.