Evaluating Equilibria

Objective Function Maximization, Pareto Efficiency, Notions of Fairness

DySPAN 2007 April 17, 2007

Optimality

- In general we assume the existence of some design objective function $J:A \rightarrow \mathbb{R}$
- The desirability of a network state, *a*, is the value of *J*(a).
- In general maximizers of *J* are unrelated to fixed points of *d*.

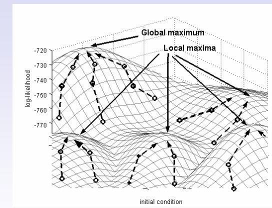


Figure from Fig 2.6 in I. Akbar, "Statistical Analysis of Wireless Systems Using Markov Models," PhD Dissertation, Virginia Tech, January 2007

Example Functions

- Utilitarian
  - Sum of all players' utilities
  - Product of all players' utilities
- Practical
  - Total system throughput
  - Average SINR
  - Maximum End-to-End Latency
  - Minimal sum system interference
- Objective can be unrelated to utilities

Utilitarian Maximizers

Γ	*N*	*W*
n	(9.6,9.6)	(3.2,21)
w	(21,3.2)	(7,7)

System Throughput Maximizers

Γ	*N*	*W*
n	(9.6,9.6)	(3.2,21)
w	(21,3.2)	(7,7)

Interference Minimization

Γ	*N*	*W*
n	(9.6,9.6)	(3.2,21)
w	(21,3.2)	(7,7)

Price of Anarchy (Factor)

Ⓐ = Performance of Centralized Algorithm Solution / Performance of Distributed Algorithm Solution

Ⓐ $\geq$ 1

- Centralized solution always at least as good as distributed solution
  - Like ASIC is always at least as good as DSP
- Ignores costs of implementing algorithms
  - Sometimes centralized is infeasible (e.g., routing the Internet)
  - Distributed can sometimes (but not generally) be more costly than centralized

Γ	*N*	*W*
n	(9.6,9.6)	(3.2,21)
w	(21,3.2)	(7,7)

Ⓐ = $\frac{9.6}{7}$

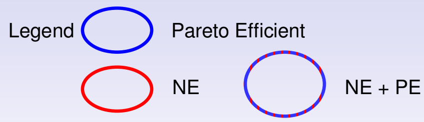
Implications

- Best of All Possible Worlds
  - Low complexity distributed algorithms with low anarchy factors
- Reality implies mix of methods
  - Hodgepodge of mixed solutions
    - Policy – bounds the price of anarchy
    - Utility adjustments – align distributed solution with centralized solution
    - Market methods – sometimes distributed, sometimes centralized
    - Punishment – sometimes centralized, sometimes distributed, sometimes both
    - Radio environment maps –"centralized" information for distributed decision processes
  - Fully distributed
    - Potential game design – really, the panglossian solution, but only applies to particular problems

Pareto efficiency (optimality)

- **Formal definition**: An action vector a^* is ***Pareto efficient*** if there exists no other action vector *a*, such that every radio's valuation of the network is at least as good and at least one radio assigns a higher valuation
- **Informal definition:** An action tuple is *Pareto efficient* if some radios must be hurt in order to improve the payoff of other radios.
- **Important note**
  - Like design objective function, unrelated to fixed points (NE)
  - Generally less specific than evaluating design objective function

Example Games



	a_2	b_2
a_1	1,1	-5,5
b_1	5,-5	-1,-1

	a_2	b_2
a_1	1,1	-5,5
b_1	5,-5	3, 3

Notions of Fairness

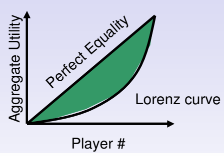
- What is "Fair"?
  - Abstractly "fair" means different things to different analysts
  - In every day life, really just short hand for "I deserve more than I got"
- Nonetheless is used to evaluate how equitably radio resources are distributed

Gini Coefficient

- Basic concept:
  - Order players by utility.
  - Form CDF for sorted utility distribution (Lorenz curve)
  - Integrate (sum) the difference between perfect equality (of outcome) and CDF
  - Divide result by sum of all players' utilities
- Formula

$$G(a)=\frac{1}{n}\left(n+1-2\frac{\sum_{i\in N}(n+1-i)u_i(a)}{\sum_{i\in N}u_i(a)}\right)$$

- Used in a lot of macro-economic comparisons of income distributions
- Relatively simple, independent of scale, independent of size of *N*, anonymity
- Radically different outcomes can give the same result



Γ	N	W
n	(9.6,9.6)	(3.2,21)
w	(21,3.2)	(7,7)

G	N	W
n	0	0.37
w	0.37	0

Other Metrics of Fairness

- Theill Index

$$T(a)=\frac{1}{n}\sum_{i\in N}\left(\frac{u_i(a)}{\bar{u}}\ln\frac{u_i(a)}{\bar{u}}\right) \qquad \bar{u}(a)=\frac{1}{n}\sum_{i\in N}u_i(a)$$

- Atkinson Index, ε is income inequality aversion

$$T(a)=1-\frac{1}{\bar{u}}\left(\frac{1}{n}\sum_{i\in N}u_i(a)^{1-\varepsilon}\right)^{1/(1-\varepsilon)} ,\varepsilon\in[0,1)$$

$$T(a)=1-\frac{1}{\bar{u}}\left(\frac{1}{n}\sum_{i\in N}u_i(a)\right)^{1/n} ,\varepsilon=1$$

Summary of Equilibria Evaluation

- Lots of different ways which a point can be evaluated
- Many are contradictory
- Loosely, any point could be said to be optimal given the right objective function
- Insufficient to say that a point is optimal
- Must describe the metric in use
- Suggestion: use whatever metric makes sense to you as a network designer