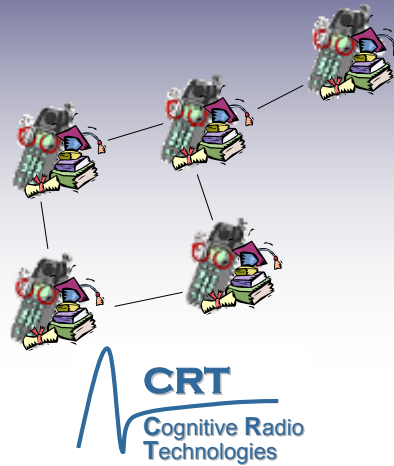# The Analysis of Cognitive Radio Networks

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SDR Forum 2008
Oct 27, 2008
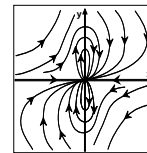
CRT
Cognitive **R**adio
Technologies

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Presentation Overview

- The interactive decision problem of cognitive radio networks

- Traditional analysis techniques

- Game theory based techniques
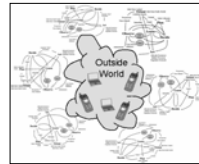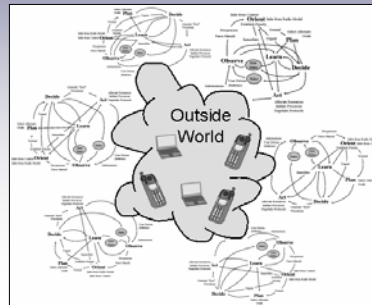
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1

The Interactive Problem with Networked Cognitive Radios

Concept, Examples, and Modeling

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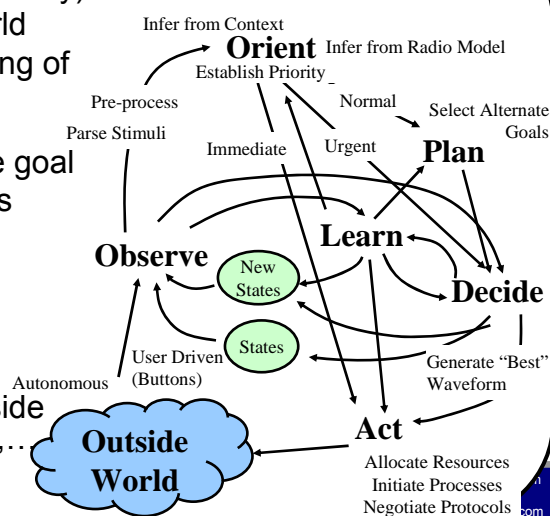

Conceptual Operation

OODA Loop: (continuously)
- Observe outside world
- Orient to infer meaning of observations
- Adjust waveform as needed to achieve goal
- Implement processes needed to change waveform

Other processes: (as needed)
- Adjust goals (Plan)
- Learn about the outside world, needs of user,…

Cognition cycle [Mitola_99]

The Interaction Problem

Outside World

- Outside world is determined by the interaction of numerous cognitive radios
- Adaptations spawn adaptations

Dynamic Spectrum Access (DSA) is a success. So what's next?

- We're confident we can detect and rapidly vacate a band when a primary user (PU) shows up
- Confidence leads to 802.22, 802.16h, 802.11h, 802.11y, White Space Proposal
- But vacating one band means you're hopping into another band
- Successful networks are capacity constrained
- So vacating a PU's band will generally mean we're interfering with some SU

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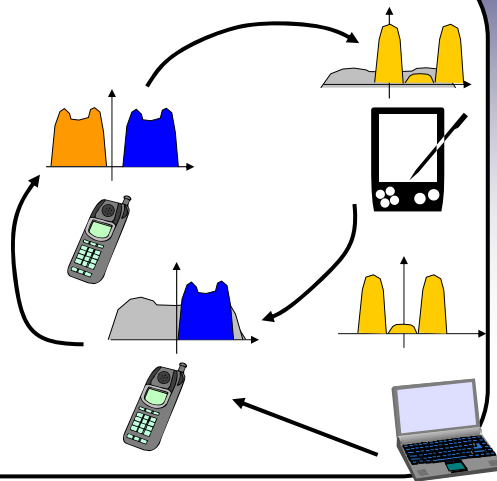
In heavily loaded networks, a single vacation can spawn an infinite adaptation process

- Suppose
  - $g_{31} > g_{21}$; $g_{12} > g_{32}$; $g_{23} > g_{13}$
- Without loss of generality
  - g_{31}, g_{12}, g_{23} = 1
  - g_{21}, g_{32}, g_{13} = 0.5
- Infinite Loop!
  - 4,5,1,3,2,6,4,…

Phone Image:
http://www1.istockphoto.com/file_thumbview_approve/2820949/2/istockphoto_2820949_dect_phone.jpg
Cradle Image:
http://www.skypejournal.com/blog/archives/images/AVM_7170_D.jpg
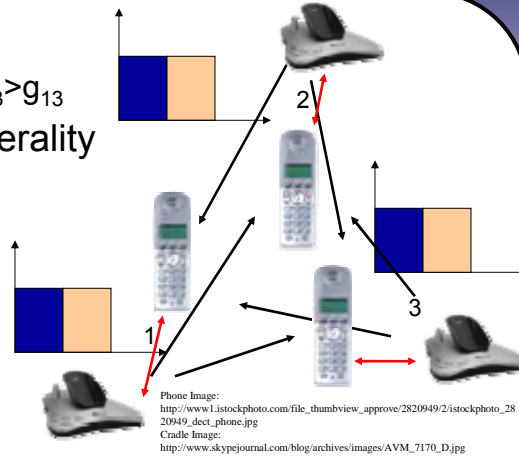
Interference Characterization

Chan.	(0,0,0)	(0,0,1)	(0,1,0)	(0,1,1)	(1,0,0)	(1,0,1)	(1,1,0)	(1,1,1)
Interf.	(1.5,1.5,1.5)	(0.5,1,0)	(1,0,0.5)	(0,0.5,1)	(0,0.5,1)	(1,0,0.5)	(0.5,1,0)	(1.5,1.5,1.5)
	0	1	2	3	4	5	6	7

Generalized Insights from the DECT Example

- If # allocations > # channels, non-centralized DSA will have a non-zero probability of looping
- As # allocations $\rightarrow \infty$, probability of looping goes to 1
- Can be mitigated by increasing # of channels (DECT has 120) or reducing frequency of adaptations (DECT is every 30 minutes)
  - Both waste spectrum
  - And we're talking 100's of ms for vacation times
- "Centralized" solutions become distributed as networks scale
  - "Rippling" in Cisco WiFi Enterprise Networks
    - www.hubbert.org/labels/Ripple.html

Locally optimal decisions that lead to globally undesirable networks

- Scenario: Distributed SINR maximizing power control in a single cluster
- For each link, it is desirable to increase transmit power in response to increased interference
- Steady state of network is all nodes transmitting at maximum power

Power

SINR
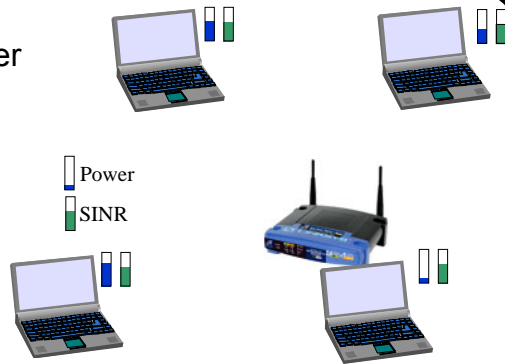
Insufficient to consider only a single link, must consider interaction

---

Potential Problems with Networked Cognitive Radios

Distributed	**Centralized**
• Infinite recursions	• Signaling Overhead
• Instability (chaos)	• Complexity
• Vicious cycles	• Responsiveness
• Adaptation collisions	• Single point of failure
• Equitable distribution of resources	
• Byzantine failure	
• Information distribution	

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Network Analysis Objectives
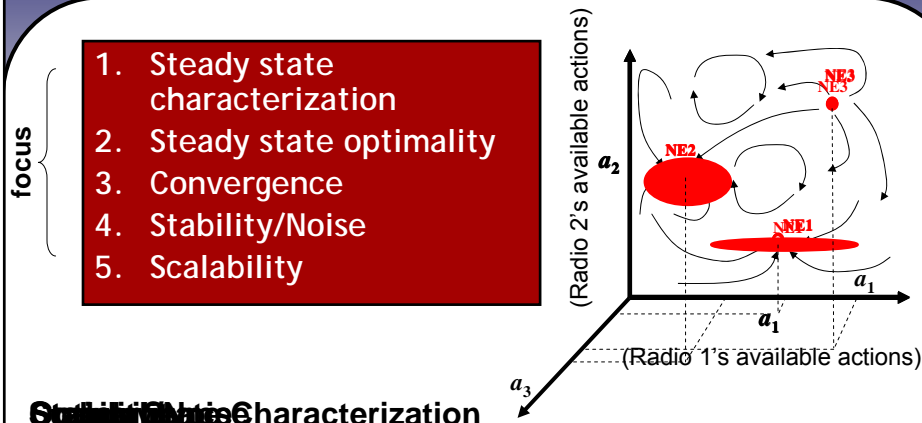
1. Steady state characterization
2. Steady state optimality
3. Convergence
4. Stability/Noise
5. Scalability

focus

(Radio 2's available actions)

a_2

NE3

NE2

NE1

a_1

a_1

(Radio 1's available actions)

a_3

Steady State Characterization

(overlapping text, illegible)

Why focus on OODA loop, i.e., why exclude other levels?

- OODA loop is implemented now (possibly just ODA loop as little work on context awareness)
- Changing plans
  - Over short intervals plans don't change
  - Messy in the general case (work could easily accommodate better response equivalent goals)
- Negotiating
  - Could be analyzed, but protocols fuzzy
  - General case left for future work

- Learning environment
  - Implies improving observations/orientation. Over short intervals can be assumed away
  - Left for future work
- Creation of new actions, new goals, new decision rules makes analysis impossible
  - Akin to solving a system of unknown functions of unknown variables
  - Most of this learning is supposed to occur during "sleep" modes
    - Won't be observed during operation

General Model (Focus on OODA Loop Interactions)

- Cognitive Radios

- Set *N*
- Particular radios, *i, j*

Outside World
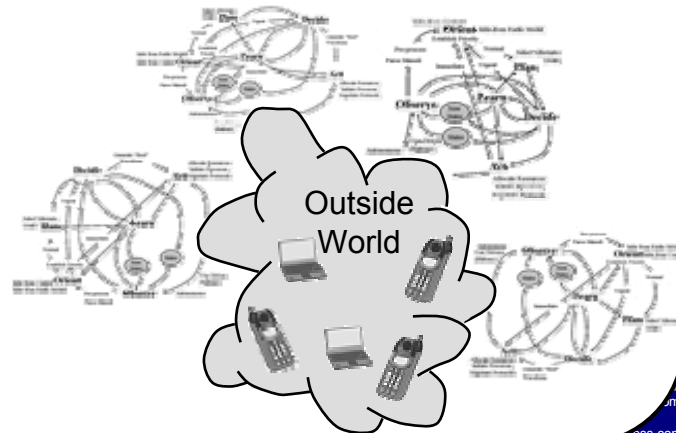
General Model (Focus on OODA Loop Interactions)

Actions

- Different radios may have different capabilities
- May be constrained by policy
- Should specify each radio's available actions to account for variations
- Actions for radio *i*
  - A_i

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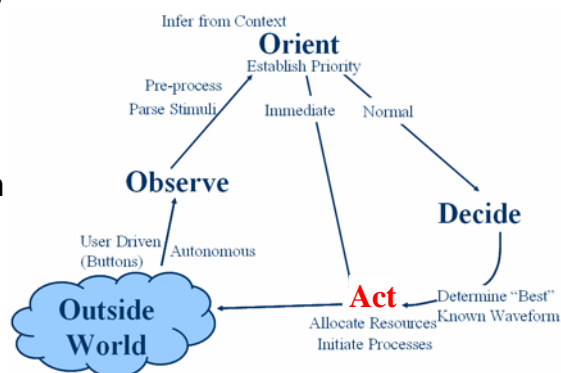
Actions

A_i – Set of available actions for radio i

a_i – A particular action chosen by i, $a_i \in A_i$

A – Action Space, Cartesian product of all A_i

$A = A_1 \times A_2 \times \cdots \times A_n$

a – Action tuple – a point in the Action Space

A_{-i} – Another action space A formed from

$A_{-i} = A_1 \times A_2 \times \cdots \times A_{i-1} \times A_{i+1} \times \cdots \times A_n$

a_{-i} – A point from the space A_{-i}

$A = A_i \times A_{-i}$

Example Two Radio Action Space

$A_1 = A_2 = [0 \ \infty)$

$A = A_1 \times A_2$
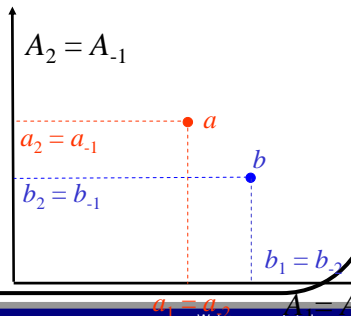
$A_2 = A_{-1}$

$a_2 = a_{-1}$

$b_2 = b_{-1}$

$b_1 = b_2$

$a_1 = a_2 \quad A = A_2$

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General Model (Focus on OODA Loop Interactions)

Decision Rules

- Maps observations to actions
  - $d_i : O \rightarrow A_i$
- Intelligence implies that these actions further the radio's goal
  - $u_i : O \rightarrow \Re$
- The many different ways of doing this merit further discussion

Implies very simple, deterministic function, e.g., standard interference function
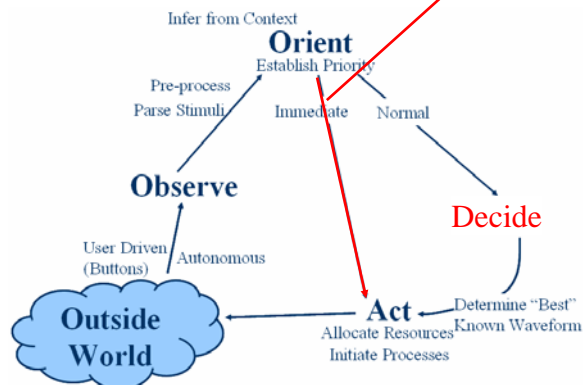
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Modeling Interactions (1/3)

Radio 1

Actions
Decision
Rules

Radio 2

Actions
Decision
Rules

Action Space

Informed by
Communications
Theory

$f : A \rightarrow O$

Outcome Space

u_1

$u_1(\hat{\gamma}_1)$

$u_2(\hat{\gamma}_2)$

u_2

$(\hat{\gamma}_1, \hat{\gamma}_2)$
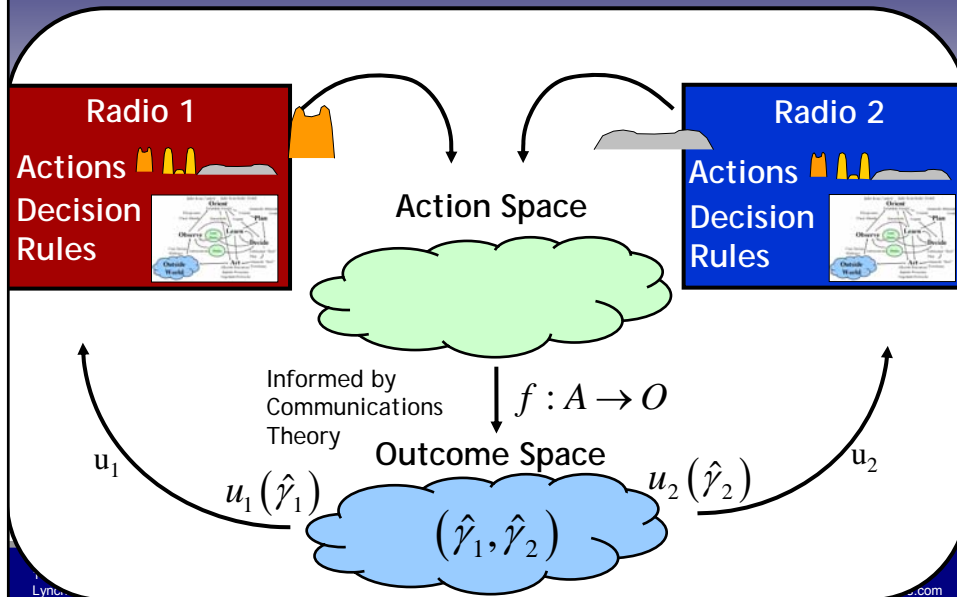
Modeling Interactions (2/3)

- Radios implement actions, but observe outcomes.
- Sometimes the mapping between outcomes and actions is one-to-one implying f is invertible.
- In this case, we can express goals and decision rules as functions of action space.
  - Simplifies analysis
- One-to-one assumption invalid in presence of noise.
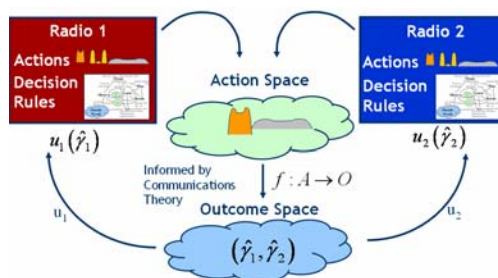
Modeling Interactions (3/3)

- When decisions are made also matters and different radios will likely make decisions at different time
- T_j – when radio j makes its adaptations
  - Generally assumed to be an infinite set
  - Assumed to occur at discrete time
    - Consistent with DSP implementation
- $T = T_1 \cup T_2 \cup \cdots \cup T_n$
- $t \in T$

Decision timing classes
- Synchronous
  - All at once
- Round-robin
  - One at a time in order
  - Used in a lot of analysis
- Random
  - One at a time in no order
- Asynchronous
  - Random subset at a time
  - Least overhead for a network

Cognitive Radio Network Modeling Summary

- Radios
- Actions for each radio
- Observed Outcome Space
- Goals
- Decision Rules
- Timing

- $i,j \in N,\ |N| = n$
- $A = A_1 \times A_2 \times \cdots \times A_n$
- O

- $u_j : O \rightarrow \Re\ (u_j : A \rightarrow \Re)$
- $d_j : O \rightarrow A_i\ (d_j : A \rightarrow A_i)$
- $T = T_1 \cup T_2 \cup \cdots \cup T_n$

Symbol	Meaning	Symbol	Meaning
N	Set of cognitive radios	i, j	Particular cognitive radios
A_j	Adaptations for j	a_j	Adaptation chosen by j
a_{-j}	Adaptation vector excluding a_j	u_j	Goal of j
O	Set of outcomes	O_j	Outcome observed by j
d_j	Decision rule for j	T_j	Times when j adapts
T	Adaptation times $\forall j \in N$	t	An element of T

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DFS Example

- Two radios
- Two common channels
  - Implies 4 element action space
- Both try to maximize Signal-to-Interference Ratio
- Alternate adaptations

General Model Symbols	Modeled System Parameters
N (cognitive radio set)	$\{1,2\}$
A (action space)	$\left\{ \left(\omega_{1_a},\omega_{2_a}\right), \left(\omega_{1_a},\omega_{2_b}\right), \left(\omega_{1_b},\omega_{2_a}\right), \left(\omega_{1_b},\omega_{2_b}\right) \right\}$
$\{u_j\}$ (utility functions)	$u_j(a) = \dfrac{g_j}{g_{-j}\left\lvert \rho\left(\omega_j, \omega_{-j}\right)\right\rvert}$
$\{d_j\}$ (decision rules)	$d_j(a) = \underset{\omega_j \in \left(w_{j_a}, \omega_{j_b}\right)}{\arg\max}\, u_j(a)$
T_j (decision timings)	$T_2 - 0.5\text{s} = T_1 = \mathbb{N}$

Items to Remember

- Cognitive radios introduce interactive decision problems
- When studying a cognitive radio network should identify
  - Who are the decision makers
  - Available adaptations of the decision makers
  - Goals guiding the decision makers
  - Rules being used to formulate decisions
  - Any timing information

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Traditional Approaches to Modeling and Analysis
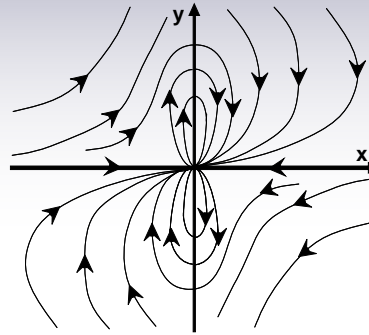
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Outline

- Concepts:
  - Dynamical Systems Model
  - Fixed Points
  - Optimality
  - Convergence
  - Stability
- Models
  - Contraction Mappings
  - Markov chains
  - Standard Interference Function

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Basic Model

- *Dynamical system*
  - A system whose change in state is a function of the current state and time
- Autonomous system
  - Not a function of time
  - OK for synchronous timing
- Characteristic function

$$\dot{a} = g(a,t)$$

- Evolution function

$$d : A \times T \to A$$

  - First step in analysis of dynamical system
  - Describes state as function of time & initial state.
  - For simplicity

$$d = \underset{j \in N}{\times} d_j = d : A \to A$$
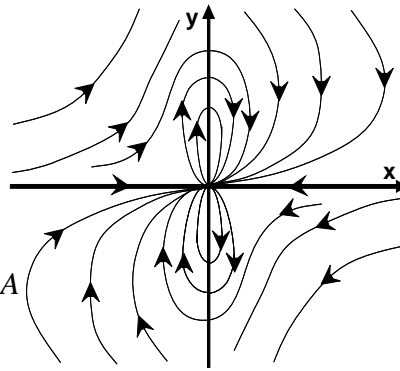
while noting the relevant timing model

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Connection to Cognitive Radio Model

- *g* = $\Delta d / \Delta t$ (generally discrete time for our purposes)
- Assumption of a known decision rule obviates need to solve for evolution function.
- Reflects innermost loop of the OODA loop
- Useful for deterministic procedural radios

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Differences with CRN model

- Goals of secondary importance
  - Technically not needed
- Not appropriate for ontological radios
  - May not be a closed form expression for decision rule and thus no evolution function
  - Really only know that radio will "intelligently" – work towards its goal
- Unwieldy for random procedural radios
  - Possible to model as Markov chain, but requires empirical work or very detailed analysis to discover transition probabilities

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Steady-states

- Recall model of $<N,A,\{d_i\},T>$ which we characterize with the evolution function d
- Steady-state is a point where $a^* = d(a^*)$ for all $t \geq t^*$
- Obvious solution: solve for fixed points of d.
- For non-cooperative radios, if a^* is a fixed point under synchronous timing, then it is under the other three timings.
- Works well for convex action spaces
  - Not always guaranteed to exist
  - Value of fixed point theorems
- Not so well for finite spaces
  - Generally requires exhaustive search

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Fixed Point Definition

Given a mapping $d : A \to A$ a point $a^* \in A$ is said to be a fixed point of d if $d(a^*) = a^*$

In 2-D fixed points for d can be found by evaluating where $b = d(a)$ and $b = a$ intersect.

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Optimality

- In general we assume the existence of some design objective function $J{:}A{\to}\mathbb{R}$

- The desirableness of a network state, *a*, is the value of *J*(a).

- In general maximizers of *J* are unrelated to fixed points of *d*.

Figure from Fig 2.6 in I. Akbar, "Statistical Analysis of Wireless Systems Using Markov Models," PhD Dissertation, Virginia Tech, January 2007

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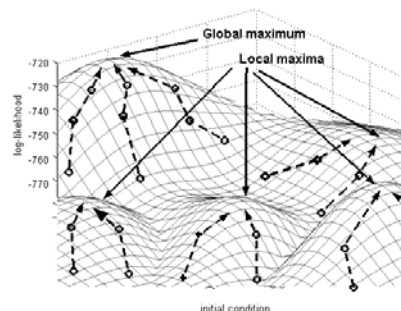
Identification of Optimality

- If *J* is differentiable, then optimal point must either lie on a boundary or be at a point where the gradient is the zero vector

$$\nabla J(a) = \frac{\partial J(a)}{\partial a_1}\hat{a}_1 + \frac{\partial J(a)}{\partial a_2}\hat{a}_2 + \cdots + \frac{\partial J(a)}{\partial a_n}\hat{a}_n = \mathbf{0}$$
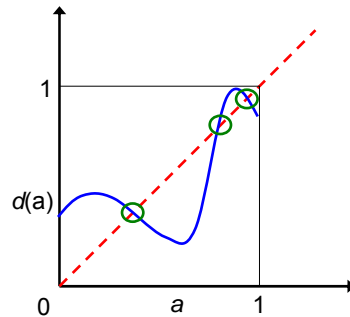
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Convergent Sequence

- A sequence $\{p_n\}$ in a Euclidean space *X* with point $p \in X$ such that for every $\varepsilon > 0$, there is an integer *N* such that $n \geq N$ implies $d_X(p_n, p) < \varepsilon$

- This can be equivalently written as $\lim_{n \to \infty} p_n = p$ or $p_n \to p$

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Example Convergent Sequence

$$p_n = 1/n$$

- Given ε, choose $N = 1/\varepsilon$, $p = 0$

0   1

Establish convergence by applying definition
Necessitates knowledge of p.

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Showing convergence with nonlinear programming

Theorem 5.30: *Zangwill's Convergence Theorem A* [Zangwill_69]
Let $d{:}A \rightarrow A$ determine an algorithm that given a point a^0 generates a sequence $\left(a^k \right)_0^\infty$ through the iteration $a^{k+1} \in d(a^k)$. Let a solution set, $S^* \subset A$, be given. Suppose
(1) All points $\left(a^k \right)_0^\infty$ are in a compact set $S \subset A$.
(2) There is a *continuous function* $\alpha : A \rightarrow \mathbb{R}$ such that:
   (a) if $a \notin S^*$, then $\alpha(a') > \alpha(a) \forall a' \in d(a)$
   (b) if $a \in S^*$, then $\alpha(a') \geq \alpha(a) \forall a' \in d(a)$
(3) d is closed at a if $a \notin S^*$.
Then either the recursion $a^{k+1} \in d(a^k)$ arrives at a solution (fixed point), or the limit of any convergent subsequence of $\left(a^k \right)_0^\infty$ is in S^*.
Proof: A proof of this theorem is given in [Zangwill_69].
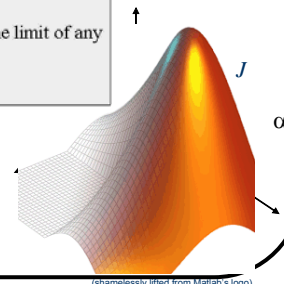
J

α

Left unanswered: where does α come from?
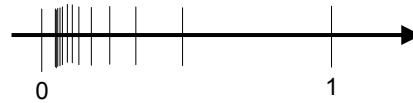
(shamelessly lifted from Matlab's logo)

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Stability

Definition 3.5: *Lyapunov stability*
We say that an action vector, a^*, is *Lyapunov stable* if for every $\varepsilon > 0$ there is a $\delta > 0$ such that for all $t \geq t^0$, $\left\| a(t^0), a^* \right\| < \delta \Rightarrow \left\| a(t), a^* \right\| < \varepsilon$ [8].

Definition 3.6: *Attractvity*
The action vector a^* is said to be *attractive* over the region $S \subset A$, $S = \left\{ a \in A \mid \left\| a, a^*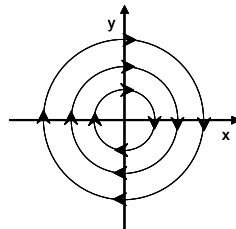 \right\| < M \right\}$, if given any $a(t_0) \in S$, the sequence $\left\{ a(t) \right\}$ converges to a^* for $t \geq t_0$.

Stable, but not attractive
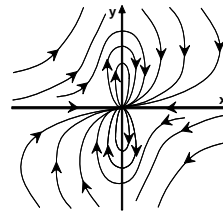
Attractive, but not stable

Lyapunov's Direct Method

Theorem 3.3: *Lyapunov's Direct Method for Discrete Time Systems*
Given a recursion $a(t^{k+1}) = d^I(a(t^k))$ with fixed point a^*, we know that a^* is Lyapunov stable if there exists a continuous function (known as a **Lyapunov function**) that maps a neighborhood of a^* to the real numbers, i.e., $L : N(a^*) \to \mathbb{R}$, such that the following three conditions are satisfied:

1) $L(a^*) = 0$

2) $L(a) > 0 \ \forall a \in N(a^*) \setminus a^*$

3) $\Delta L(a(t)) \equiv L\left[d^I(a(t)) \right] - L(a(t)) \leq 0 \ \forall a \in N(a^*) \setminus a^*$

Further, if conditions 1-3 hold and

a) $N(a^*) = A$, then a^* is globally Lyapunov stable;

b) $\Delta L(a(t)) < 0 \ \forall a \in N(a^*) \setminus a^*$, then a^* is asymptotically stable;

c) $N(a^*) = A$ and $\Delta L(a(t)) < 0 \ \forall a \in N(a^*) \setminus a^*$, then a^* is globally asymptotically stable.

Left unanswered: where does *L* come from?

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Analysis models appropriate for dynamical systems

- Contraction Mappings
  - Identifiable unique steady-state
 - Everywhere convergent, bound for convergence rate
  - Lyapunov stable ($\delta=\epsilon$)
    - Lyapunov function = distance to fixed point
  - General Convergence Theorem (Bertsekas) provides convergence for asynchronous timing if contraction mapping under synchronous timing
- Standard Interference Function
  - Forms a pseudo-contraction mapping
  - Can be applied beyond power control
- Markov Chains (Ergodic and Absorbing)
  - Also useful in game analysis
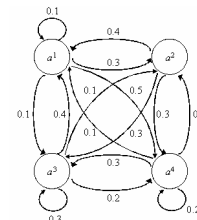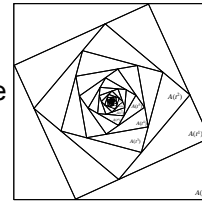
Contraction Mappings

Definition 3.8: *Contraction mapping*
A decision rule, d, is said to be a *contraction mapping* with modulus α if there is an $\alpha \in [0,1)$ such that $\|d(a)-d(b)\| \le \alpha \|a-b\| \; \forall b,a \in A$.

Definition 3.9: *Pseudo-contraction*
Given mapping $d : A \to A$ with fixed point, a^*, we say d is a *pseudo-contraction* if there is an $\alpha \in [0,1)$ such that $\|d(a)-d(a^*)\| \le \alpha \|a-a^*\| \; \forall a \in A$.

- Every contraction is a pseudo-contraction
- Every pseudo-contraction has a fixed point
- Every pseudo-contraction converges at a rate of α

$$d\big(a(t),a^*\big) \le \alpha^t d\big(a(0),a^*\big)$$

- Every pseudo-contraction is globally asymptotically stable
  - Lyapunov function is distance to the fixed point)
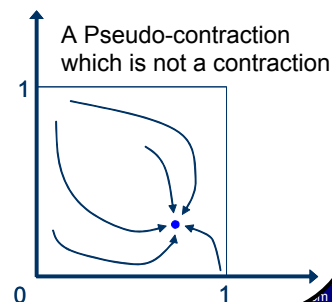
A Pseudo-contraction which is not a contraction

Standard Interference Function

- Conditions
- Suppose $d: A \rightarrow A$ and d satisfies:
  - <u>Positivity</u>: $d(a) > 0$
  - <u>Monotonicity</u>: If $a^1 \geq a^2$, then $d(a^1) \geq d(a^2)$
  - <u>Scalability</u>: For all $\alpha > 1$, $\alpha d(a) > d(\alpha\, a)$
- d is a pseudo-contraction mapping [Berggren] under synchronous timing
  - Implies synchronous and asynchronous convergence
  - Implies stability

R. Yates, "A Framework for Uplink Power Control in Cellular Radio Systems," *IEEE JSAC.*, Vol. 13, No 7, Sep. 1995, pp. 1341-1347.
F. Berggren, "Power Control, Transmission Rate Control and Scheduling in Cellular Radio Systems," PhD Dissertation Royal Institute of Technology, Stockholm, Sweden, May, 2001.

Yates' power control applications

- Target SINR algorithms

$$\gamma_j = \frac{g_{jj} p_j}{\sum_{k \in N} g_{kj} p_j + N_j} \qquad p_j^{k+1} = p_j^k \frac{\hat{\gamma}_j}{\gamma_j}$$

- <u>Fixed assignment</u> - each mobile is assigned to a particular base station
- <u>Minimum power assignment</u> - each mobile is assigned to the base station in the network where its SINR is maximized
- <u>Macro diversity</u> - all base stations in the network combine the signals of the mobiles
- <u>Limited diversity</u> - a subset of the base stations combine the signals of the mobiles
- <u>Multiple connection reception</u> - the target SINR must be maintained at a number of base stations.

Example steady-state solution

- Consider Standard Interference Function

$$p_j^{m+1} = p_j^m \frac{\hat{\gamma}_j}{\gamma_j} \qquad p_j^{m+1} = p_j^m \frac{\hat{\gamma}_j \left(\sum_{k \in N \setminus i} g_{kj} p_k^m + \sigma_j \right)}{K g_{jj} p_j^m}$$

$$p_j^* = \frac{\hat{\gamma}_j}{K g_{jj}} \left(\sum_{k \in N \setminus i} g_{kj} p_k^* + \sigma_j \right)$$

$$\begin{bmatrix} K g_{11}/\hat{\gamma}_1 & -g_{12} & \cdots & -g_{1n} \\ -g_{21} & K g_{22}/\hat{\gamma}_2 & & \\ \vdots & & \ddots & \\ -g_{n1} & -g_{n2} & \cdots & K g_{nn}/\hat{\gamma}_n \end{bmatrix} \mathbf{p}^* = \begin{bmatrix} \sigma_1 & \sigma_2 & \cdots & \sigma_n \end{bmatrix}$$

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Markov Chains

- Describes adaptations as probabilistic transitions between network states.
  - *d* is nondeterministic
- Sources of randomness:
  - Nondeterministic timing
  - Noise
- Frequently depicted as a weighted digraph or as a transition matrix
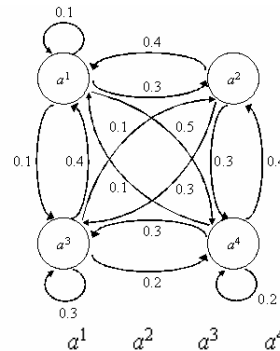
$$\mathbf{P} = \begin{array}{c} \\ \alpha^1 \\ \alpha^2 \\ \alpha^3 \\ \alpha^4 \end{array} \begin{array}{cccc} \alpha^1 & \alpha^2 & \alpha^3 & \alpha^4 \\ \begin{bmatrix} 0.1 & 0.3 & 0.1 & 0.5 \\ 0.4 & 0.0 & 0.3 & 0.3 \\ 0.4 & 0.1 & 0.3 & 0.2 \\ 0.1 & 0.4 & 0.3 & 0.2 \end{bmatrix} \end{array}$$

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General Insights [Stewart_94]

- Probability of occupying a state after two iterations.
  - Form **PP**.
  - Now entry p^{mn} in the mth row and n^{th} column of **PP** represents the probability that system is in state a^n two iterations after being in state a^m.
- Consider P^k.
  - Then entry p^{mn} in the mth row and n^{th} column of represents the probability that system is in state a^n two iterations after being in state a^m.

$$\mathbf{P} = \begin{array}{c} a^1 \\ a^2 \\ a^3 \\ a^4 \end{array} \begin{array}{cccc} a^1 & a^2 & a^3 & a^4 \\ \left[0.1 \right. & 0.3 & 0.1 & 0.5 \\ 0.4 & 0.0 & 0.3 & 0.3 \\ 0.4 & 0.1 & 0.3 & 0.2 \\ 0.1 & 0.4 & 0.3 & \left. 0.2 \right] \end{array}$$

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Steady-states of Markov chains

- May be inaccurate to consider a Markov chain to have a fixed point
  - Actually ok for absorbing Markov chains
- Stationary Distribution
  - A probability distribution such that π^* such that $\pi^{*T} \mathbf{P} = \pi^{*T}$ is said to be a stationary distribution for the Markov chain defined by **P**.
- Limiting distribution
  - Given initial distribution π^0 and transition matrix **P**, the *limiting distribution* is the distribution that results from evaluating $\lim_{k \to \infty} \pi^{0T} \mathbf{P}^k$

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Ergodic Markov Chain

- [Stewart_94] states that a Markov chain is ergodic if it is a Markov chain if it is a) *irreducible*, b) *positive recurrent*, and c) *aperiodic*.
- Easier to identify rule:
  - For some k $\mathbf{P}^k$ has only nonzero entries
- (Convergence, steady-state) If ergodic, then chain has a unique limiting stationary distribution.

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Absorbing Markov Chains

- Absorbing state
  - Given a Markov chain with transition matrix **P**, a state a^m is said to be an absorbing state if $p^{mm}=1$.
- Absorbing Markov Chain
  - A Markov chain is said to be an *absorbing Markov chain* if
    - it has at least one absorbing state and
    - from every state in the Markov chain there exists a sequence of state transitions with nonzero probability that leads to an absorbing state. These nonabsorbing states are called *transient states*.

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Absorbing Markov Chain Insights ([Kemeny_60])

- Canonical Form

$$\mathbf{P}' = \left[\begin{array}{c|c} \mathbf{Q} & \mathbf{R} \\ \hline \mathbf{0} & \mathbf{I}^{ab} \end{array} \right]$$

- Fundamental Matrix

$$\mathbf{N} = \left(\mathbf{I} - \mathbf{Q} \right)^{-1}$$

- Expected number of times that the system will pass through state a^m given that the system starts in state a^k.
  - n^{km}
- (Convergence Rate) Expected number of iterations before the system ends in an absorbing state starting in state a^m is given by t^m where 1 is a ones vector
  - **t=N1**
- (Final distribution) Probability of ending up in absorbing state a^m given that the system started in a^k is b^{km} where

$$\mathbf{B} = \mathbf{NR}$$

Two-Channel DFS

Decision Rule

$$d_j \left(f_j, f_{-j} \right) = \begin{cases} f_j & u_j(a) = 1 \\ f \in F \setminus f_j & u_j(a) = -1 \end{cases}$$

Goal

$$u_j(a) = \begin{cases} 1 & f_j \neq f_{-j} \\ -1 & f_j = f_{-j} \end{cases}$$

Timing

Random timer set to go off with probability p=0.5 at each iteration
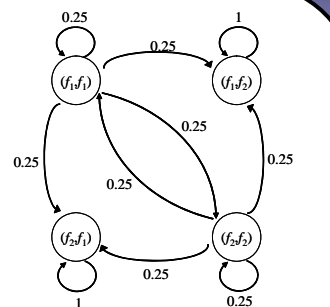
	$(f_1 f_1)$	$(f_1 f_2)$	$(f_2 f_1)$	$(f_2 f_2)$
$(f_1 f_1)$	0.25	0.25	0.25	0.25
$\mathbf{P} = (f_1 f_2)$	0	1	0	0
$(f_2 f_1)$	0	0	1	0
$(f_2 f_2)$	0.25	0.25	0.25	0.25

	$(f_1 f_1)$	$(f_2 f_2)$
$\mathbf{N} = (f_1 f_1)$	1.5	0.5
$(f_2 f_2)$	0.5	1.5

	$(f_1 f_2)$	$(f_2 f_1)$
$\mathbf{B} = (f_1 f_1)$	0.5	0.5
$(f_2 f_2)$	0.5	0.5

Comments on "Traditional" Techniques

- Perhaps a bit of a stretch to call it "traditional" with respect to cognitive radios
- Fixed point theorems provide little insight into convergence or stability
- Applying definitions to analysis can be tedious
- Models can speed up analysis
- Contraction mappings rarely encountered
  - Do apply to important class of power control algorithms
- Traditional techniques do not directly address nondeterministic algorithms
  - Empirically construct Markov models
- No help if all you have is the cognitive radios' goal and actions
  - Perhaps common from a regulator's perspective
  - What happens if they innovate a new algorithm?
  - Or if algorithm is adapted based on conditions?

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A Whirlwind Review of Game Theory

Normal form games, Nash equilibria, Pareto efficiency, Improvement Paths, Noise
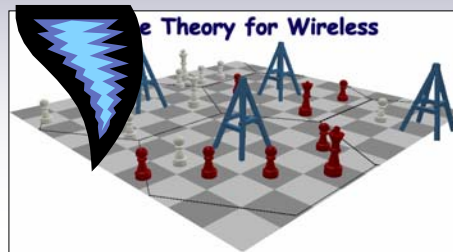
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Game

1. A (well-defined) set of 2 or more players
2. A set of actions for each player.
3. A set of preference relationships for each player for each possible action tuple.

- More elaborate games exist with more components but these three must always be there.
- Some also introduce an outcome function which maps action tuples to outcomes which are then valued by the preference relations.
- Games with just these three components (or a variation on the preference relationships) are said to be in **Normal** form or **Strategic** Form

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Set of Players (decision makers)

- N – set of n players consisting of players "named" {1, 2, 3,…,i, j,…,n}
- Note the n does not mean that there are 14 players in every game.
- Other components of the game that "belong" to a particular player are normally indicated by a subscript.
- Generic players are most commonly written as i or j.
- Usage: N is the SET of players, n is the number of players.
- $N \setminus i = \{1,2,…,i\text{-}1, i\text{+}1 ,…, n\}$ All players in N except for i

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Preference Relations (1/2)

Preference Relation expresses an individual player's desirability of one outcome over another (A binary relationship)

$\succsim_i$ Preference Relationship (prefers at least as much as)

$$o \succsim_i o^* \quad o \text{ is preferred at least as much as } o^* \text{ by player } i$$

$\succ_i$ Strict Preference Relationship (prefers strictly more than)

$$o \succ_i o^* \text{ iff } o \succsim_i o^* \text{ but not } o^* \succsim_i o$$

$\sim_i$ "Indifference" Relationship (prefers equally)

$$o \sim_i o^* \text{ iff } o \succsim_i o^* \text{ and } o^* \succsim_i o$$

Preference Relationship (2/2)

- Games generally assume the relationship between actions and outcomes is invertible so preferences can be expressed over action vectors.
- Preferences are really an *ordinal* relationship
  - Know that player prefers one outcome to another, but quantifying by how much introduces difficulties

Utility Functions (1/2) (Objective Fcns, Payoff Fcns)

A mathematical description of preference relationships.

Maps action space to set of real numbers.

$$u_i : A \rightarrow \mathbb{R}$$

Preference Relation then defined as

$$a \succsim_i a^* \text{ iff } u_i(a) \geq u_i(a^*)$$

$$a \succ_i a^* \text{ iff } u_i(a) > u_i(a^*)$$

$$a \sim_i a^* \text{ iff } u_i(a) = u_i(a^*)$$

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Utility Functions (2/2)

By quantifying preference relationships all sorts of valuable mathematical operations can be introduced.

Also note that the quantification operation is not unique as long as relationships are preserved. Many map preference relationships to [0,1].

Example

Jack prefers Apples to Oranges

$$Apples \succ_{Jack} Oranges \iff u_{Jack}(Apples) > u_{Jack}(Oranges)$$

a) $u_{Jack}(Apples) = 1$, $u_{Jack}(Oranges) = 0$

b) $u_{Jack}(Apples) = -1$, $u_{Jack}(Oranges) = -7.5$

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Variety of game models

- Normal Form Game $<N,A,\{u_i\}>$
  - Synchronous play
  - T is a singleton
  - Perfect knowledge of action space, other players' goals (called utility functions)
- Repeated Game $<N,A,\{u_i\},\{d_i\}>$
  - Repeated synchronous play of a normal form game
  - T may be finite or infinite
  - Perfect knowledge of action space, other players' goals (called utility functions)
  - Players may consider actions in future stages and current stages
    - Strategies (modified d_i)
- Asynchronous myopic repeated game $<N,A,\{u_i\},\{d_i\},T>$
  - Repeated play of a normal form game under various timings
  - Radios react to most recent stage, decision rule is "intelligent"
- Many others in the literature and in the dissertation

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Cognitive radios are naturally modeled as players in a game
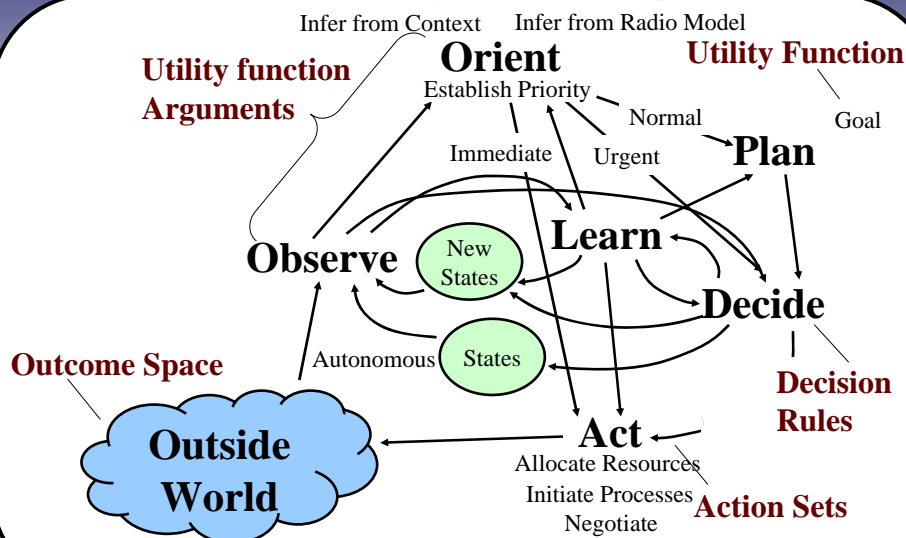
Adapted From Mitola, "Cognitive Radio for Flexible Mobile Multimedia Communications ", IEEE Mobile Multimedia Conference, 1999, pp 3-10.

Interaction is naturally modeled as a game

Radio 1

Actions
Decision
Rules

Radio 2

Actions
Decision
Rules

Action Space

Informed by Communications Theory

$f : A \rightarrow O$

Outcome Space

u_1

$u_1(\hat{\gamma}_1)$

$u_2(\hat{\gamma}_2)$

u_2

$(\hat{\gamma}_1, \hat{\gamma}_2)$
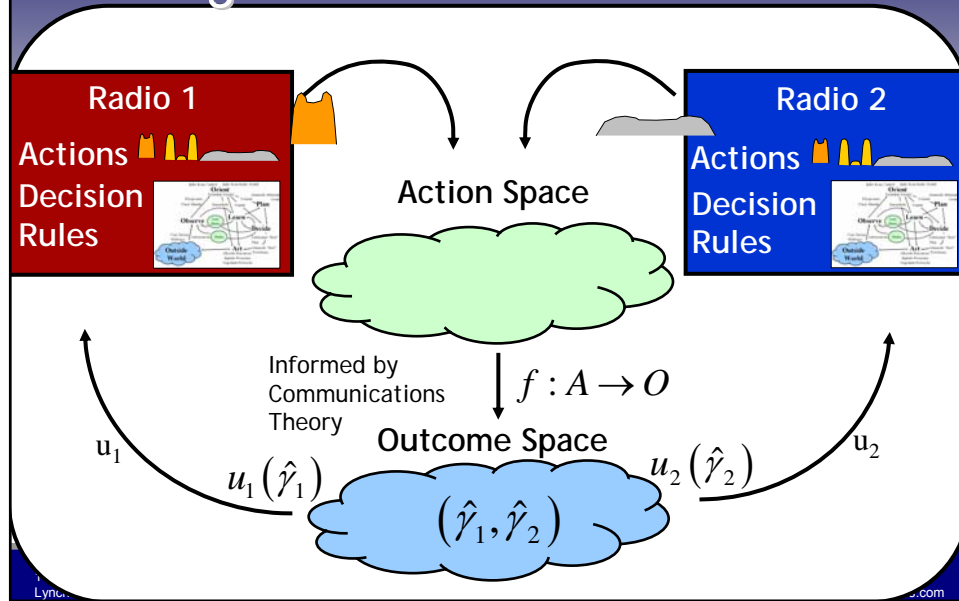
Some differences between game models and cognitive radio network model

- Assuming numerous iterations, normal form game only has a single stage.
  - Useful for compactly capturing modeling components at a single stage
  - Normal form game properties will be exploited in the analysis of other games

	Player	Cognitive Radio
Knowledge	Knows A	Can learn O (may know or learn A)
$f : A \rightarrow O$	Invertible	Not invertible (noise)
	Constant	May change over time (though relatively fixed for short periods)
	Known	Has to learn
Preferences	Ordinal	Cardinal (goals)

Steady-states

- Recall model of $<N,A,\{d_i\},T>$ which we characterize with the evolution function d
- Steady-state is a point where $a^* = d(a^*)$ for all $t \geq t^*$
- Obvious solution: solve for fixed points of d.
- For non-cooperative radios, if a^* is a fixed point under synchronous timing, then it is under the other three timings.
- Works well for convex action spaces
  - Not always guaranteed to exist
  - Value of fixed point theorems
- Not so well for finite spaces
  - Generally requires exhaustive search

Nash Equilibrium

"A steady-state where each player holds a correct expectation of the other players' behavior and acts rationally." - Osborne
An action vector from which no player can profitably unilaterally deviate.

Definition

An action tuple a is a NE if for every $i \in N$ $u_i(a_i, a_{-i}) \geq u_i(b_i, a_{-i})$

for all $b_i \in A_i$.

Note showing that a point is a NE says nothing about the process by which the steady state is reached. Nor anything about its uniqueness nor its stability.
Also note that we are implicitly assuming that only pure strategies are possible in this case.

Examples

- Cognitive Radios' Dilemma
  - Two radios have two signals to choose between {n,w} and {N,W}
  - n and N do not overlap
  - Higher throughput from operating as a high power wideband signal when other is narrowband
- Jamming Avoidance
  - Two channels
  - No NE

Γ	N	W
n	(9.6,9.6)	(3.2,21)
w	(21,3.2)	(7,7)

Jammer

Transmitter		0	1
	0	(-1,1)	(1,-1)
	1	(1,-1)	(-1,1)

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Nash Equilibrium as a Fixed Point

- Individual Best Response

$$\hat{B}_i(a) = \left\{ b_i \in A_i : u_i(b_i, a_{-i}) \ge u_i(a_i, a_{-i}) \, \forall a_i \in A_i \right\}$$

- Synchronous Best Response

$$\hat{B}(a) = \underset{i \in N}{\times} \hat{B}_i(a)$$

- Nash Equilibrium as a fixed point

$$a^* = \hat{B}(a^*)$$

- Fixed point theorems can be used to establish existence of NE (see dissertation)
- NE can be solved by implied system of equations

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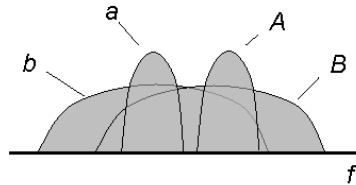147 Mill Ridge Rd, Ste 119
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Best Response Analysis

Goal

$$u_i(c) = \left(B - \sum_{k \in N} c_k \right) c_i - K c_i$$

Best Response

$$c_i = \hat{B}_i(c) = \left(B - K - \sum_{k \in N \setminus i} c_k \right) / 2$$

Simultaneous System of Equations

c_1	$+$	$0.5\,c_2$	$+$	$0.5\,c_3$	$+$	$0.5\,c_4$	$+$	$0.5\,c_5$	$=$	$(B\text{-}K)/2$
$0.5\,c_1$	$+$	c_2	$+$	$0.5\,c_3$	$+$	$0.5\,c_4$	$+$	$0.5\,c_5$	$=$	$(B\text{-}K)/2$
$0.5\,c_1$	$+$	$0.5\,c_2$	$+$	c_3	$+$	$0.5\,c_4$	$+$	$0.5\,c_5$	$=$	$(B\text{-}K)/2$
$0.5\,c_1$	$+$	$0.5\,c_2$	$+$	$0.5\,c_3$	$+$	c_4	$+$	$0.5\,c_5$	$=$	$(B\text{-}K)/2$
$0.5\,c_1$	$+$	$0.5\,c_2$	$+$	$0.5\,c_3$	$+$	$0.5\,c_4$	$+$	c_5	$=$	$(B\text{-}K)/2$

Solution

$$\hat{c}_i = (B - K)/6 \ \forall i \in N$$

Generalization

$$\hat{c}_i = (B - K)/(|N| + 1) \ \forall i \in N$$

Significance of NE for CRNs

Theorem 4.1: NE and Cognitive Radio Network Steady States (*)

Given cognitive radio network $\langle N, A, \{u_i\}, \{d_i\}, T \rangle$ where all players are autonomously rational, if the game $\langle N, A, \{u_i\} \rangle$ has an NE a^*, then a^* is a fixed point for d.

Proof: Suppose a^* is not a fixed point. Then for some $i \in N$, there must be some $b_i \in d_i(a^*)$ with $b_i \neq a_i^*$ such that $u_i(b_i, a_{-i}^*) > u_i(a_i^*, a_{-i}^*)$. But this contradicts the assumption that a^* is an NE. Therefore, a^* must be a fixed point for d.

Autonomously Rational Decision Rule

$$b_i \in d_i(a), \ b_i \neq a_i \ \Rightarrow \ u_i(b_i, a_{-i}) > u_i(a_i, a_{-i})$$

- Why not "if and only if"?
  - Consider a self-motivated game with a local maximum and a hill-climbing algorithm.
  - For many decision rules, NE do capture all fixed points (see dissertation)
- Identifies steady-states for all "intelligent" decision rules with the same goal.
- Implies a mechanism for policy design while accommodating differing implementations
  - Verify goals result in desired performance
  - Verify radios act intelligently
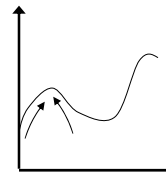
Optimality

- In general we assume the existence of some design objective function $J: \mathrm{A} \rightarrow \mathbb{R}$
- The desirableness of a network state, *a*, is the value of $J(a)$.
- In general maximizers of *J* are unrelated to fixed points of *d*.
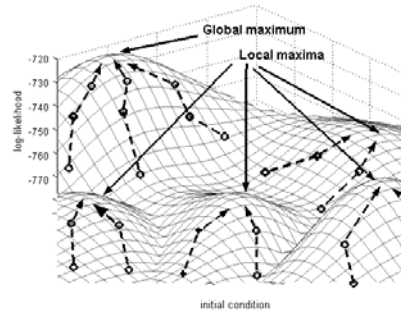
Figure from Fig 2.6 in I. Akbar, "Statistical Analysis of Wireless Systems Using Markov Models," PhD Dissertation, Virginia Tech, January 2007

Example Functions

- Utilitarian
  - Sum of all players' utilities
  - Product of all players' utilities
- Practical
  - Total system throughput
  - Average SINR
  - Maximum End-to-End Latency
  - Minimal sum system interference
- Objective can be unrelated to utilities

Utilitarian Maximizers

Γ	N	W
n	(9.6,9.6)	(3.2,21)
w	(21,3.2)	(7,7)

System Throughput Maximizers

Γ	N	W
n	(9.6,9.6)	(3.2,21)
w	(21,3.2)	(7,7)

Interference Minimization

Γ	N	W
n	(9.6,9.6)	(3.2,21)
w	(21,3.2)	(7,7)

Price of Anarchy (Factor)

$$\mathcal{A} = \frac{\text{Performance of Centralized Algorithm Solution}}{\text{Performance of Distributed Algorithm Solution}}$$

$$\mathcal{A} \geq 1$$

- Centralized solution always at least as good as distributed solution
  - Like ASIC is always at least as good as DSP
- Ignores costs of implementing algorithms
  - Sometimes centralized is infeasible (e.g., routing the Internet)
  - Distributed can sometimes (but not generally) be more costly than centralized

Γ	N	W
n	(9.6,9.6)	(3.2,21)
w	(21,3.2)	(7,7)

$$\mathcal{A} = \frac{9.6}{7}$$

Implications

- Best of All Possible Worlds
  - Low complexity distributed algorithms with low anarchy factors
- Reality implies mix of methods
  - Hodgepodge of mixed solutions
    - Policy – bounds the price of anarchy
    - Utility adjustments – align distributed solution with centralized solution
    - Market methods – sometimes distributed, sometimes centralized
    - Punishment – sometimes centralized, sometimes distributed, sometimes both
    - Radio environment maps –"centralized" information for distributed decision processes
  - Fully distributed
    - Potential game design – really, the panglossian solution, but only applies to particular problems

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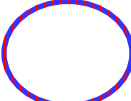
Pareto efficiency (optimality)
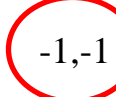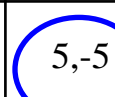
- **Formal definition**: An action vector a^* is ***Pareto efficient*** if there exists no other action vector a, such that every radio's valuation of the network is at least as good and at least one radio assigns a higher valuation
- **Informal definition:** An action tuple is *Pareto efficient* if some radios must be hurt in order to improve the payoff of other radios.
- **Important note**
  - Like design objective function, unrelated to fixed points (NE)
  - Generally less specific than evaluating design objective function

Example Games

Legend ⬭ Pareto Efficient

⬭ NE   ⬭ NE + PE

	a_2	b_2
a_1	1,1	-5,5
b_1	5,-5	-1,-1

	a_2	b_2
a_1	1,1	-5,5
b_1	5,-5	3, 3

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The Notion of Time and Imperfections in Games and Networks

Extensive Form Games, Repeated Games, Convergence Concepts in Normal Form Games, Trembling Hand Games, Noisy Observations

Model Timing Review

- When decisions are made also matters and different radios will likely make decisions at different time
- T_j – when radio j makes its adaptations
  - Generally assumed to be an infinite set
  - Assumed to occur at discrete time
    - Consistent with DSP implementation
- $T = T_1 \cup T_2 \cup \cdots \cup T_n$
- $t \in T$

Decision timing classes
- Synchronous
  - All at once
- Round-robin
  - One at a time in order
  - Used in a lot of analysis
- Random
  - One at a time in no order
- Asynchronous
  - Random subset at a time
  - Least overhead for a network

Repeated Games

- Same game is *repeated*
  - Indefinitely
  - Finitely
- Players consider discounted payoffs across multiple stages
  - Stage k
  $$\tilde{u}_i\left(a^k\right) = \delta^k u_i\left(a^k\right)$$
  - Expected value over all future stages
  $$\widehat{u}_i\left(\left(a^k\right)\right) = \sum_{k=0}^{\infty} \delta^k u_i\left(a^k\right)$$

Stage 1

Γ	N	W
n	(9.6,9.6)	(3.2,21)
w	(21,3.2)	(7,7)

Stage 2

Γ	N	W
n	(9.6,9.6)	(3.2,21)
w	(21,3.2)	(7,7)

Stage k

Γ	N	W
n	(9.6,9.6)	(3.2,21)
w	(21,3.2)	(7,7)

Myopic Processes

- Players have no knowledge about utility functions, or expectations about future play, typically can observe or infer current actions
- Best response dynamic – maximize individual performance presuming other players' actions are fixed
- Better response dynamic – improve individual performance presuming other players' actions are fixed
- Interesting convergence results can be established

Paths and Convergence

- *Path* [Monderer_96]
  - A path in Γ is a sequence $\gamma = (a^0, a^1,\ldots)$ such that for every $k \geq 1$ there exists a unique player such that the strategy combinations (a^{k-1}, a^k) differs in exactly one coordinate.
  - Equivalently, a path is a sequence of unilateral deviations. When discussing paths, we make use of the following conventions.
  - Each element of γ is called a *step*.
  - a^0 is referred to as the *initial* or *starting point* of γ.
  - Assuming γ is finite with m steps, a^m is called the *terminal point* or *ending point* of γ and say that γ has *length m*.
- *Cycle* [Voorneveld_96]
  - A finite path $\gamma = (a^0, a^1,\ldots,a^k)$ where $a^k = a^0$

Improvement Paths

- Improvement Path
  - A path $\gamma = (a^0, a^1,\ldots)$ where for all $k\geq 1$, $u_i(a^k) > u_i(a^{k-1})$ where i is the unique deviator at k

- Improvement Cycle
  - An improvement path that is also a cycle
  - See the DFS example

	A	γ_1	B	
a	(5,5)	γ_5	(-1,10)	γ_4
b γ_2	(10,-1) γ_6		(0,0)	
		γ_3		

$\gamma_1 = ((a,A),(a,B))$	$\gamma_3 = ((b,A),(b,B))$	$\gamma_5 = (\gamma_1,(b,B))$
$\gamma_2 = ((a,A),(b,A))$	$\gamma_4 = ((a,B),(b,B))$	$\gamma_6 = (\gamma_1,(b,B))$

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Convergence Properties

- Finite Improvement Property (FIP)
  - All improvement paths in a game are finite
- Weak Finite Improvement Property (weak FIP)
  - From every action tuple, there exists an improvement path that terminates in an NE.
- FIP implies weak FIP
- FIP implies lack of improvement cycles
- Weak FIP implies existence of an NE

Examples

Game with FIP

	A	B
a	1,-1	0,2
b	-1,1	2,2

Weak FIP but not FIP

	A	B	C
a	1,-1	-1,1	0,2
b	-1,1	1,-1	1,2
c	2,0	2,1	2,2

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Implications of FIP and weak FIP

- Assumes radios are incapable of reasoning ahead and must react to internal states and current observations
- Unless the game model of a CRN has weak FIP, then no autonomously rational decision rule can be guaranteed to converge from all initial states under random and round-robin timing (Theorem 4.10 in dissertation).
- If the game model of a CRN has FIP, then ALL autonomously rational decision rules are guaranteed to converge from all initial states under random and round-robin timing.
  - And asynchronous timings, but not immediate from definition
- More insights possible by considering more refined classes of decision rules and timings

Decision Rules

Definition 4.10: *Best Response Dynamic*
A decision rule $d_i : A \to A_i$ is a best response dynamic if each adaptation would maximize the radio's utility if all other radios continued to implement the same waveforms, i.e., $d_i(a) \in \{b_i \in A_i : u_i(b_i, a_{-i}) \geq u_i(a_i, a_{-i}) \forall a_i \in A_i\}$

Definition 4.11: *Better Response Dynamic*
A decision rule $d_i : A \to A_i$ is a better response dynamic if each adaptation would improve the radio's utility if all other radios continued to implement the same waveforms, i.e., $d_i(a) \in \{b_i \in A_i : u_i(b_i, a_{-i}) > u_i(a_i, a_{-i})\}$.

Definition 4.13: *Friedman's Random Better Response* [Friedman_01]
Player *i* chooses an action from $A_i \backslash b_i$ where b_i is player *i*'s current action according to a uniform random distribution. If the chosen action would improve the utility of player *i*, it is implemented, otherwise, the player continues to play b_i.

Definition 4.12: *Random Better Response Dynamic* (*)
A decision rule $d_i : A \to A_i$ is a random better response dynamic if for each $t_i \in T_i$, radio *i* chooses an action from A_i where each action has a nonzero probability of being chosen and implements the action if it would improve its utility.

Absorbing Markov Chains and Improvement Paths

- Sources of randomness
  - Timing (Random, Asynchronous)
  - Decision rule (random decision rule)
  - Corrupted observations
- An NE is an absorbing state for autonomously rational decision rules.
- Weak FIP implies that the game is an absorbing Markov chain as long as the NE terminating improvement path always has a nonzero probability of being implemented.
- This then allows us to characterize
  - convergence rate,
  - probability of ending up in a particular NE,
  - expected number of times a particular transient state will be visited

Connecting Markov models, improvement paths, and decision rules

- Suppose we need the path $\gamma = (a^0, a^1, \ldots a^m)$ for convergence by weak FIP.
- Must get right sequence of players and right sequence of adaptations.
- Friedman Random Better Response
  - Random or Asynchronous
    - Every sequence of players have a chance to occur
    - Random decision rule means that all improvements have a chance to be chosen
  - Synchronous not guaranteed

Γ	A	B
a	(1,1)	(0,0)
b	(0,0)	(1,1)

- Alternate random better response (chance of choosing same action)
  - Because of chance to choose same action, every sequence of players can result from every decision timing.
  - Because of random choice, every improvement path has a chance of occurring

Convergence Results (Finite Games)

Decision Rules	Timings			
	Round-Robin	Random	Synchronous	Asynchronous
Best Response	1,3	1,3	1	1,3
Exhaustive Better Response	3	3	-	3
Random Better Response[a]	1,2,3	1,2,3	1,2,3	1,2,3
Random Better Response[b]	1,3	1,2,3	1	1,2,3

(a) Definition 4.12, (b) Definition 4.13, 1. IESDS, 2. Weak FIP, 3. FIP

- If a decision rule converges under round-robin, random, or synchronous timing, then it also converges under asynchronous timing.
- Random better responses converge for the most decision timings and the most surveyed game conditions.
  - Implies that non-deterministic procedural cognitive radio implementations are a good approach if you don't know much about the network.

Trembling Hand ("Noise" in Games)

- Assumes players have a nonzero chance of making an error implementing their action.
  - Who has not accidentally handed over the wrong amount of cash at a restaurant?
  - Who has not accidentally written a "tpyo"?
- Related to errors in observation as erroneous observations cause errors in implementation (from an outside observer's perspective).

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Noisy decision rules

- Noisy utility $\tilde{u}_i(a,t) = u_i(a) + n_i(a,t)$

Trembling Hand

Definition 4.20: *Friedman's Noisy Random Better Response* [Friedman_01]
Player i chooses an action $a_i \in A_i \backslash b_i$ where b_i is player i's current action according to a uniform random distribution. If $u_i(a_i, a_{-i}) > u_i(b_i, a_{-i})$, then a_i is implemented, however, if $u_i(a_i, a_{-i}) \le u_i(b_i, a_{-i})$, then player i still switches to a_i with nonzero probability ρ.

Definition 4.21: *Noisy Best Response Dynamic* (*)
A decision rule $\tilde{d}_i : A \times T \to A_i$ is a noisy best response dynamic if each adaptation would maximize the radio's noisy utility if all other radios continued to implement the same waveforms, i.e., $\tilde{d}_i(a) \in \{b_i \in A_i : \tilde{u}_i(b_i, a_{-i}, t) \ge \tilde{u}_i(a_i, a_{-i}, t) \forall a_i \in A_i\}$

Observation Errors

Definition 4.22: *Noisy Better Response Dynamic* (*)
A decision rule $\tilde{d}_i : A \times T \to A_i$ is a noisy better response dynamic if each adaptation would improve the radio's utility if all other radios continued to implement the same waveforms, i.e., $\tilde{d}_i(a) \in \{b_i \in A_i : \tilde{u}_i(b_i, a_{-i}, t) > \tilde{u}_i(a_i, a_{-i}, t)\}$.

Definition 4.23: *Noisy Random Better Response Dynamic* (*)
A decision rule $\tilde{d}_i : A \times T \to A_i$ is a random better response dynamic if for each $t_i \in T_i$, radio i chooses an action from A_i with nonzero probability and implements the action if it would improve $\tilde{u}_i$.

Implications of noise

- For random timing, [Friedman] shows game with noisy random better response is an ergodic Markov chain.
- Likewise other observation based noisy decision rules are ergodic Markov chains
  - Unbounded noise implies chance of adapting (or not adapting) to any action
  - If coupled with random, synchronous, or asynchronous timings, then CRNs with corrupted observation can be modeled as ergodic Makov chains.
  - Not so for round-robin (violates aperiodicity)
- Somewhat disappointing
  - No real steady-state (though unique limiting stationary distribution)

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DFS Example with three access points

- 3 access nodes, 3 channels, attempting to operate in band with least spectral energy.
- Constant power
- Link gain matrix

g_{ik}	1	2	3
1	1	0.5	0.1
2	0.5	1	0.3
3	0.1	0.3	1

- Noiseless observations

(f_1,f_1,f_1)	(f_1,f_1,f_2)	(f_1,f_2,f_1)	(f_1,f_2,f_2)	(f_2,f_1,f_1)	(f_2,f_1,f_2)	(f_2,f_2,f_1)	(f_2,f_2,f_2)	(f_2,f_1,f_1)
$(0.6,0.8,0.4)$	$(0.5,0.5,0.0)$	$(0.1,0.0,0.1)$	$(0.0,0.3,0.3)$	$(0.0,0.3,0.3)$	$(0.1,0.0,0.1)$	$(0.5,0.5,0.0)$	$(0.6,0.8,0.4)$	$(0.0,0.3,0.3)$

- Random timing

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Trembling Hand

- Transition Matrix, p=0.1

P	(f_1,f_1,f_1)	(f_1,f_1,f_2)	(f_1,f_2,f_1)	(f_1,f_2,f_2)	(f_2,f_1,f_1)	(f_2,f_1,f_2)	(f_2,f_2,f_1)	(f_2,f_2,f_2)
(f_1,f_1,f_1)	0	1/3	1/3	0	1/3	0	0	0
(f_1,f_1,f_2)	1/30	3/10	0	1/3	0	1/3	0	0
(f_1,f_2,f_1)	1/30	0	9/10	1/30	0	0	1/30	0
(f_1,f_2,f_2)	0	1/30	1/3	3/5	0	0	0	1/30
(f_2,f_1,f_1)	1/30	0	0	0	3/5	1/3	1/30	0
(f_2,f_1,f_2)	0	1/30	0	0	1/30	9/10	0	1/30
(f_2,f_2,f_1)	0	0	1/3	0	1/3	0	3/10	1/30
(f_2,f_2,f_2)	0	0	0	1/3	0	1/3	1/3	0

- Limiting distribution

(f_1,f_1,f_1)	(f_1,f_1,f_2)	(f_1,f_2,f_1)	(f_1,f_2,f_2)	(f_2,f_1,f_1)	(f_2,f_1,f_2)	(f_2,f_2,f_1)	(f_2,f_2,f_2)
0.0161	0.0293	0.3846	0.0699	0.0699	0.3846	0.0293	0.0161

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Noisy Best Response

- Transition Matrix, $\mathcal{N}(0,1)$ Gaussian Noise

P	$(f_1f_1f_1)$	$(f_1f_1f_2)$	$(f_1f_2f_1)$	$(f_1f_2f_2)$	$(f_2f_1f_1)$	$(f_2f_1f_2)$	$(f_2f_2f_1)$	$(f_2f_2f_2)$
$(f_1f_1f_1)$	0.3367	0.2038	0.2381	0	0.2214	0	0	0
$(f_1f_1f_2)$	0.1295	0.4813	0	0.1854	0	0.2038	0	0
$(f_1f_2f_1)$	0.0953	0	0.6273	0.1479	0	0	0.1295	0
$(f_1f_2f_2)$	0	0.1479	0.1854	0.5548	0	0	0	0.1119
$(f_2f_1f_1)$	0.1119	0	0	0	0.5548	0.1854	0.1479	0
$(f_2f_1f_2)$	0	0.1295	0	0	0.1479	0.6273	0	0.0953
$(f_2f_2f_1)$	0	0	0.2038	0	0.1854	0	0.4813	0.1295
$(f_2f_2f_2)$	0	0	0	0.2214	0	0.2381	0.2038	0.3367

- Limiting stationary distributions

	$(f_1f_1f_1)$	$(f_1f_1f_2)$	$(f_1f_2f_1)$	$(f_1f_2f_2)$	$(f_2f_1f_1)$	$(f_2f_1f_2)$	$(f_2f_2f_1)$	$(f_2f_2f_2)$
$\sigma=1.00$	0.0709	0.1120	0.1765	0.1406	0.1406	0.1765	0.1120	0.0709
$\sigma=0.50$	0.0540	0.1040	0.1984	0.1436	0.1436	0.1984	0.1040	0.0540
$\sigma=0.10$	0.0129	0.0647	0.2857	0.1366	0.1366	0.2857	0.0647	0.0129
$\sigma=0.05$	0.0033	0.0397	0.3387	0.1183	0.1183	0.3387	0.0397	0.0033
$\sigma=0.01$	0	0.002	0.46	0.038	0.038	0.46	0.002	0

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Comment on Noise and Observations

- Cardinality of goals makes a difference for cognitive radios
  - Probability of making an error is a function of the difference in utilities
  - With ordinal preferences, utility functions are just useful fictions
    - Might as well assume a trembling hand
- Unboundedness of noise implies that no state can be absorbing for most decision rules
- NE retains significant predictive power
  - While CRN is an ergodic Markov chain, NE (and the adjacent states) remain most likely states to visit
  - Stronger prediction with less noise
  - Also stronger when network has a Lyapunov function
  - Exception - elusive equilibria ([Hicks_04])

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Items to Remember

Game	⇔	Cognitive radio network
Player	⇔	Cognitive radio
Actions	⇔	Actions
Utility function	⇔	Goal
Outcome space	⇔	Outside world
Utility function arguments	⇔	Observations/orientation
Order of play	⇔	Adaptation timings

- NE are always fixed points for self-interested adaptations
  - But may not be ALL fixed points
- Many ways to measure optimality
- Randomness helps convergence
- Unbounded noise implies network has a theoretically non-zero chance to visit every possible state

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Potential Games

The best game model for designing cognitive radio networks since…..

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Potential Games

- Existence of a function (called the potential function, V), that reflects the change in utility seen by a unilaterally deviating player.
- Cognitive radio interpretation:
  - Every time a cognitive radio unilaterally adapts in a way that furthers its own goal, some real-valued function increases.

$V(a)$ (vertical axis)

time (horizontal axis)

Potential Game	Relationship ($\forall i \in N, \forall a \in A$)
Exact (EPG)	$u_i(b_i,a_{-i}) - u_i(a_i,a_{-i}) = V(b_i,a_{-i}) - V(a_i,a_{-i})$
Weighted (WPG)	$u_i(b_i,a_{-i}) - u_i(a_i,a_{-i}) = \alpha_i\left[V(b_i,a_{-i}) - V(a_i,a_{-i})\right]$
Ordinal (OPG)	$u_i(b_i,a_{-i}) - u_i(a_i,a_{-i}) > 0 \Leftrightarrow V(b_i,a_{-i}) - V(a_i,a_{-i}) > 0$
Generalized Ordinal (GOPG)	$u_i(b_i,a_{-i}) - u_i(a_i,a_{-i}) > 0 \Rightarrow V(b_i,a_{-i}) - V(a_i,a_{-i}) > 0$
Generalized ε (GεPG)	$u_i(b_i,a_{-i}) > u_i(a_i,a_{-i}) + \varepsilon_1 \Rightarrow V(b_i,a_{-i}) > V(a_i,a_{-i}) + \varepsilon_2$

EPG, WPG, GεPG, OPG, GOPG

Exact Potential Game Forms

- Many exact potential games can be recognized by the form of the utility function

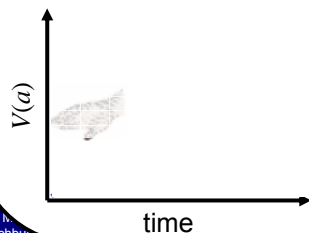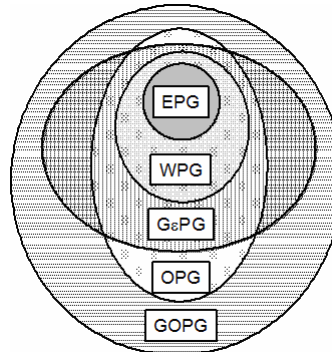| Game	Utility Function Form	Potential Function
Coordination Game	$u_i(a) = C(a)$	$V(a) = C(a)$
Dummy Game	$u_i(a) = D_i(a_{-i})$	$V(a) = c, c \in \mathbb{R}$
Coordination-Dummy Game	$u_i(a) = C(a) + D_i(a_{-i})$	$V(a) = C(a)$
Self-Motivated Game	$u_i(a) = S_i(a_i)$	$V(a) = \sum_{i \in N} S_i(a_i)$
Bilateral Symmetric Interaction (BSI) Game	$u_i(a) = \sum_{j \in N\setminus\{i\}} w_{ij}(a_i,a_j) - S_i(a_i)$ where $w_{ij}(a_i,a_j) = w_{ji}(a_j,a_i)$	$V(a) = \sum_{i \in N}\sum_{j=1}^{i-1} w_{ij}(a_i,a_j) - \sum_{i \in N} S_i(a_i)$
Multilateral Symmetric Interaction (MSI) Game	$u_i(a) = \sum_{\{S \in 2^N : i \in S\}} w_{S,i}(a_S) + D_i(a_{-i})$ where $w_{S,i}(a_S) = w_{S,j}(a_S)\, \forall i, j \in S$	$V(a) = \sum_{S \in 2^N} w_S(a_S)$

Implications of Monotonicity

- Monotonicity implies
  - Existence of steady-states (maximizers of V)
  - Convergence to maximizers of V for numerous combinations of decision timings decision rules – all self-interested adaptations
- Does not mean that that we get good performance
  - Only if V is a function we want to maximize

Decision Rules	Timings			
	Round-Robin	Random	Synchronous	Asynchronous
Best Response	1,2,4	1,2,4	-	1,2
Exhaustive Better Response	1,2	1,2	-	1,2
Random Better Response[a]	1,2,4	1,2,4	1,2	1,2
Random Better Response[b]	1,2	1,2	-	1,2
ε-Better Response[c]	1,2,3,4	1,2,3,4	-	1,2,3
Intelligently Random Better Response	1,4	1,4	-	1,2
Directional Better Response[c]	4	4	-	-
Averaged Best Response[d]	3,4	3,4	-	-

(a) Definition 4.12, (b) Definition 4.13, (c) Convergence to an ε-NE, (d) u_i quasi-concave in a_i
1. Finite game, 2. Infinite game with FIP, 3. Infinite game with AFIP, 4. Infinite game with bounded continuous potential function (implication of D^V)

Other Potential Game Properties

- All finite potential games have FIP
- All finite games with FIP are potential games
  - Very important for ensuring convergence of distributed cognitive radio networks
- $-V$ is a is a Lyapunov function for isolated maximizers
- Stable NE solvable by maximizers of V
- Linear combination of exact potential games is an exact potential game
- Maximizer of potential game need not maximize your objective function
  - Cognitive Radios' Dilemma is a potential game

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Interference Reducing Networks

- Concept
  - Cognitive radio network is a potential game with a potential function that is negation of observed network interference
- Definition
  - A network of cognitive radios where each adaptation decreases the sum of each radio's observed interference is an *IRN*

$$\Phi(\omega) = \sum_{i \in N} I_i(\omega)$$

$\Phi(\omega)$ vs. time

- Implementation:
  - Design algorithms such that network is a potential game with $\Phi \propto -V$

Designing An IRN

- Link Bilateral Symmetric Interference (BSI) if

$$I_j(\omega_j, \omega_k, p_j, p_k) = I_k(\omega_j, \omega_k, p_j, p_k)$$
$$g_{jk} p_j \rho(\omega_j, \omega_k) = g_{kj} p_k \rho(\omega_k, \omega_j)$$
$$\forall \omega_j \in \Omega_j, \forall \omega_k \in \Omega_k$$

- Network BSI if link BSI holds for the observation metrics of all pairs of decision processes

$$I_k(\omega_j, \omega_k, p_j, p_k) = p_j g_{jk} \rho(\omega_j, \omega_k)$$

p_k **K**

g_{kj}

g_{jk}

p_j

J

$$I_j(\omega_j, \omega_k, p_j, p_k) = p_k g_{kj} \rho(\omega_j, \omega_k)$$

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Selfish adaptations reduce sum network interference when BSI holds

- Sum network interference

$$\Phi(\omega, p) = \sum_{j \in N} I_j(\omega, p)$$

- With two links and BSI

$$I_j(\omega_j^1, \omega_k, p) < I_j(\omega_j^2, \omega_k, p) \Rightarrow I_k(\omega_j^1, \omega_k, p) < I_k(\omega_j^2, \omega_k, p)$$

$$\Phi(\omega_j^1, \omega_k, p) < \Phi(\omega_j^2, \omega_k, p)$$

$$\Phi(\omega_j^2, \omega_k, p) - \Phi(\omega_j^1, \omega_k, p) = 2\left(I_j(\omega_j^2, \omega_k, p) - I_j(\omega_j^1, \omega_k, p)\right)$$

Network sees twice the benefit of the selfish adapter

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This correlation between selfish and social benefit yields desirable behavior

- Convergence
  - ***ALL*** sequences of unilateral selfish adaptations induce monotonically decreasing network interference levels
  - For finite waveform sets, completely unsynchronized adaptations form absorbing Markov chains
- Optimality of steady-states
  - Assuming exhaustive adaptations, interference minimizers are the only steady-states
- Stability
  - Sum network interference is a Lyapunov function in neighborhoods of isolated interference minimizers
  - In practice, many minimizers aren't isolated, so some hysteresis is needed
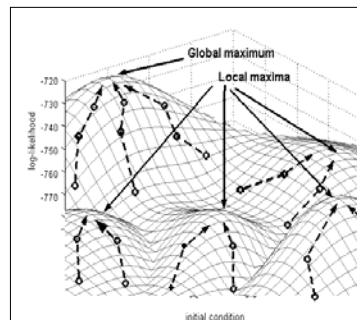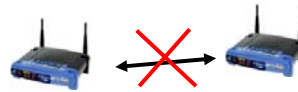

Figure from Fig 2.6 in I. Akbar, "Statistical Analysis of Wireless Systems Using Markov Models," PhD Dissertation, Virginia Tech, January 2007

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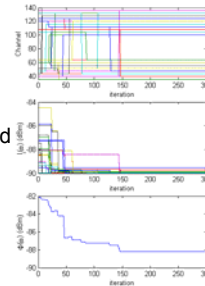147 Mill Ridge Rd, Ste 119
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This connection can be used to achieve minimal complexity

- Because selfish behavior is good for the network, no need to directly consider impact on other links
  - Means virtually no bandwidth lost to control messages
- Because selfish behavior is based solely on its own observations, there's no need to burden the network distributing observations
- Because unsynchronized adaptations converge, there no need for clock distribution
  - Will converge faster if properly synchronized
- Because *ALL* selfish adaptations converge, even trial and error, decision rules can be very simple
  - As simple as search through weighted RSSI measurements

Example unsynchronized behavior

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Situations where BSI occurs

$$g_{kj} p_k \rho\left(\omega_j, \omega_k\right) = g_{jk} p_j \rho\left(\omega_k, \omega_j\right)$$

- Isolated Network Clusters
  - All devices communicate with a common access node with identical received powers.
  - Clusters are isolated in signal space
- Close Proximity Networks
  - All devices are sufficiently close that waveform correlation effects dominate
- Controlled Observation Processes
  - Leverage knowledge of waveform protocol to create observation metrics which achieve BSI for the allowed adaptations

"I THINK YOU SHOULD BE MORE EXPLICIT HERE IN STEP TWO."

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BSI by Subtraction

- Huge number of interference sources in a network
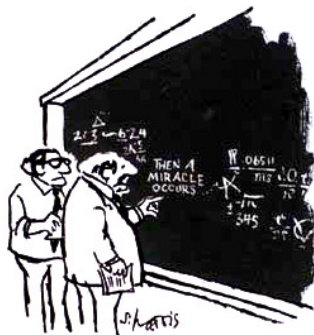- Concept: constrain observations to only consider symmetric interference sources

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An IRN 802.11 DFS Algorithm

- Suppose each **access node measures** the received signal **power** and **frequency** of the RTS/CTS (or BSSID) messages sent by observable access nodes in the network
  - Ignore client interference
- Assumed out-of-channel interference is negligible and RTS/CTS transmitted at same power

$$u_i(f) = -I_i(f) = -\sum_{k \in N \backslash i} g_{ki} p_k \sigma(f_i, f_k)$$

$$\sigma(f_i, f_k) = \begin{cases} 1 & f_i = f_k \\ 0 & f_i \neq f_k \end{cases}$$

$$g_{jk} p_j \sigma(f_j, f_k) = g_{kj} p_k \sigma(f_k, f_j)$$

Listen on Channel L_C

RTS/CTS energy detected?

Measure power of access node in message, p

Note address of access node, a

Update interference table

Start

Pick channel to listen on, L_C

Time for decision?

Apply decision criteria for new operating channel, O_C

Use 802.11h to signal change in O_C to clients

J. Neel, J. Reed, "Performance of Distributed Dynamic Frequency Selection Schemes for Interference Reducing Networks," *Milcom 2006*, Washington DC, October 23-25, 2006

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Statistics
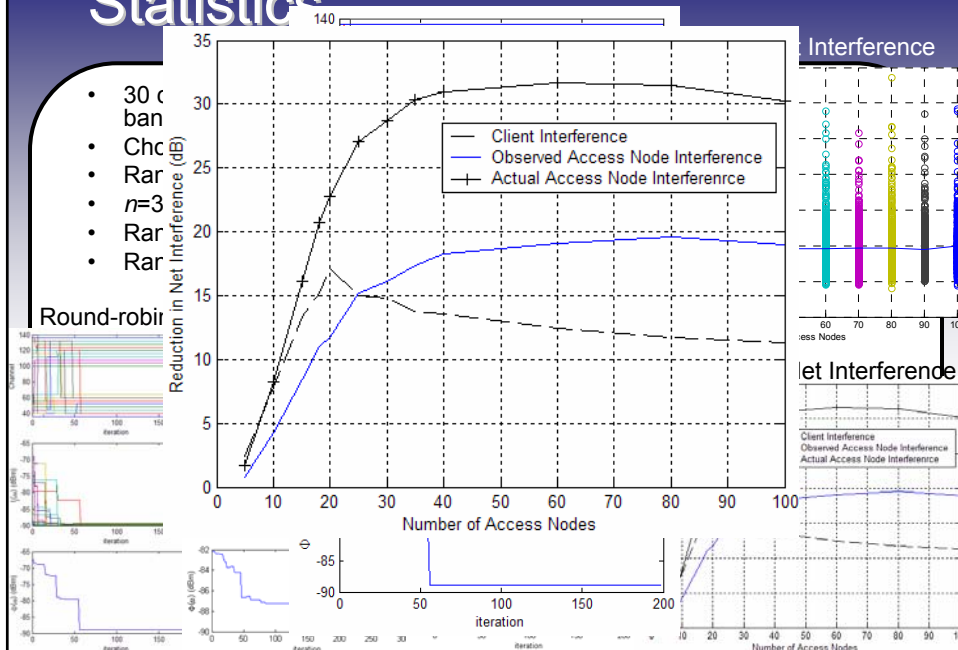
Congestion Games (1/2)

- Key components
  - a finite set of actors (players) $N=\{1,2,\dots,n\}$
  - a set of facilities, $F=\{1, 2, \dots , g\}$
  - a set of payoffs, $c_f(k)$ where and k is the number of users of facility f.
- Game Model
  - $N = N$
  - $A_i = 2^F$ (choose any subset of F)

$$u_i(a) = \sum_{f \in a_i} c_f \left(\sigma_f (a) \right)$$

  - Sum of payoffs of each facility

$$\sigma_f (a) = \#\{i \in N : f \in a_i\}$$

  - Each facility has its own function, function of # of users (anonymous)

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Congestion Games (2/2)

- Exact potential function
  - Sum of facility costs over all used facilities from 1 to # of users of each facility
- Comments:
  - Every player does not need the same action set for EPG to hold
  - Tends to either spread (costly) or concentrate (beneficial) users across facilities (as modified by club benefits)
- Examples:
  - Routers (though not with prioritization)
  - Vehicle Traffic congestion
  - Some MACs
  - Could be network selection

$$V(a) = \sum_{f \in \bigcup_{i=1}^{n} a_i} \left(\sum_{k=1}^{\sigma_f(a)} c_f(k) \right)$$

"Nobody goes there anymore. It's too crowded" -Yogi Berra

(actually John McNulty)

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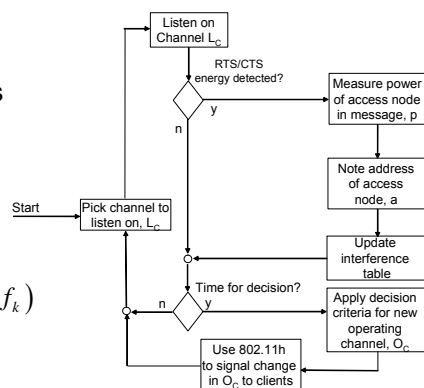Web: www.crtwireless.com
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Exact Potential Games Form a Linear Space (1/2)

Definition 5.15: *Linear Space*
Given a set X, X is said to be a *linear space* if for every $x, y, z \in X$ and every $\alpha, \beta \in \mathbb{R}$ it satisfies the following ten (10) properties:

(1) Closure under addition, $x + y \in X$
(2) Closure under scalar multiplication, i.e., $\alpha x \in X$
(3) Commutativity, i.e., $x + y = y + x$
(4) Additive Associativity, i.e., $x + y = y + x$
(5) Additive Identity, i.e., there is some $0 \in X$ such that if $x \in X$, $0 + x = X$.
(6) Additive Inverse, i.e., for every $x \in X$, there is some $-x \in X$ such that $x + (-x) = 0$.
(7) Associativity of Scalar Multiplication, i.e., $\alpha(\beta x) = (\alpha \beta) x$
(8) Distributivity of Scalar Sums, i.e., $(\alpha + \beta) x = \alpha x + \beta x$
(9) Distributivity of Vector Sums, i.e., $\alpha(x + y) = \alpha x + \alpha y$
(10) Scalar Multiplicative Identity, i.e., $1x = x$.

Theorem 5.24: *Linear Space of Exact Potential Games* [Fachini_97]
$\Gamma^{N,A}$ forms a linear space
Proof. A proof of this result is given in [Fachini_97]. However, some key aspects of this proof are repeated in the following. An additive identity element is given by the game $\Gamma = \langle N, A, \{0\} \rangle$ which has exact potential function $V(a) = 0$. Given exact potential games, $\Gamma_1, \Gamma_2 \in \Gamma^{N,A}$ with potential functions V_1 and V_2, and scalars $\alpha_1, \alpha_2 \in \mathbb{R}$, $\Gamma_3 = \alpha_1 \Gamma_1 + \alpha_2 \Gamma_2$, then Γ_3 is an exact potential game with potential $V_3 = \alpha_1 V_1 + \alpha_2 V_2$. □

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Exact Potential Games Form a Linear Space (2/2)

- Implication for design: Scale up (arbitrarily?) complex cognitive radio networks by defining radio objectives as linear combinations of simpler algorithms
  - power + frequency + routing + … ?
- **NB1**: Does not hold for weighted potential games (nor its parents)
- **NB2**: When action sets are not identical, games must either be orthogonal or <N,AxB, {u(a,b)}> and <N,AxB,{v(a,b)}>must be exact potential games

RRM + Service Allocation

- Could also combine with EPG RRM algorithms (like BSI from 12/03/07)
- Example: Right to use spectrum + interference avoidance in spectrum
  - Service Game: Two radios each own a band with different transmit policies for each band
  - RRM Game: 802.11a game from 12/03/07
  - Pay for spectrum rights + minimize interference

RRM + Service Allocation

- Service Game
  - $S_1 = B_1$, $S_2 = B_2$, $S_{3\ldots n} = \varnothing$
  - $A_{1\ldots n} = \{\varnothing, B_1, B_2, \{B_1,B_2\}\}$

- RRM / Interference Avoidance Game
  - $f_i \in \{B_1, B_2\}$

$$u_j^S(f,a) = \sum_{s_k \in a_j} b_j(s_k) - \sum_{s_k \in a_j \setminus S_j} p_{s_k}(\sigma_{s_k}(a)) + \\ + \sum_{s_j \in a_{-j}} p_{s_j}(\sigma_{s_j}(a)) - \sum_{s_j \in a} c_{s_j}(\sigma_{s_j}(a))$$

$$u_j^R(f,a) = \begin{cases} -\sum_{k \in N \setminus i} g_{ki} p_k \sigma(f_i, f_k) & f_i \in a_i \\ -c - \sum_{k \in N \setminus i} g_{ki} p_k \sigma(f_i, f_k) & f_i \notin a_i \end{cases}$$

Service Potential

$$V(a) = c + \sum_{j \in N} \sum_{s_k \in a_j} b_j(s_k)$$

$$- \sum_{s_j \in \bigcup_{i=1}^{n} a_i} \left(\sum_{m=1}^{\sigma_{s_j}(a)} \left[p_{s_j}(m) + c_{s_j}(m) \right] \right)$$

RRM Exact Potential

$$V^R(f,A) = \sum_{j \in N} band(a_j, f_j)$$

$$- \sum_{i \in N} \sum_{j=1}^{i-1} g_{ji} p_j \sigma(f_j, f_i)$$

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Using Potential Games to Design of Cognitive Radio Networks

- If we design our networks to be an exact potential games, then we can
  - Predict steady-state behavior (maximizers of *V)*
  - Know that very simple greedy algorithms will converge
  - Know that very simple algorithms will be stable
  - Scale up more complex algorithms
  - Mix different sets of algorithms
- Issues:
  - Potential function should be something we want maximized
  - Stability only holds for isolated fixed points
  - Minimize amount of external information / information exchange
- Approach
  - Find objectives that look like exact potential game utility functions that correspond
  - Look for local ways to gather information
    - Trivial to make desirable exact potential game out of coordination games
    - Possible to use other forms, may require modifying observations or defining specific network processes

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Summary of Points to Remember

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Points to Remember

- Cognitive radios introduce interactive decision problems
- When studying a cognitive radio network should identify
  - Who are the decision makers
  - Available adaptations of the decision makers
  - Goals guiding the decision makers
  - Rules being used to formulate decisions
  - Any timing information

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Comments on "Traditional" Techniques

- Perhaps a bit of a stretch to call it "traditional" with respect to cognitive radios
- Fixed point theorems provide little insight into convergence or stability
- Applying definitions to analysis can be tedious
- Models can speed up analysis
- Contraction mappings rarely encountered
    - Do apply to important class of power control algorithms
- Traditional techniques do not directly address nondeterministic algorithms
    - Empirically construct Markov models
- No help if all you have is the cognitive radios' goal and actions
    - Perhaps common from a regulator's perspective
    - What happens if they innovate a new algorithm?
    - Or if algorithm is adapted based on conditions?

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Points to Remember

Game	⇔	Cognitive radio network
Player	⇔	Cognitive radio
Actions	⇔	Actions
Utility function	⇔	Goal
Outcome space	⇔	Outside world
Utility function arguments	⇔	Observations/orientation
Order of play	⇔	Adaptation timings

- NE are always fixed points for self-interested adaptations
    - But may not be ALL fixed points
- Many ways to measure optimality
- Randomness helps convergence
- Unbounded noise implies network has a theoretically non-zero chance to visit every possible state
- Many important insights can be gained via game theory with only goals and actions
    - Some specific (NE), some more general (convergence)
- More detailed analysis possible by combining game theory with traditional techniques

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Potential Games in the Design of Cognitive Radio Networks

- If we design our networks to be an exact potential games, then we can
  - Predict steady-state behavior (maximizers of *V)*
  - Know that very simple greedy algorithms will converge
  - Know that very simple algorithms will be stable
  - Scale up more complex algorithms
  - Mix different sets of algorithms
- Issues:
  - Potential function should be something we want maximized
  - Stability only holds for isolated fixed points
  - Minimize amount of external information / information exchange
- Approach
  - Find objectives that look like exact potential game utility functions that correspond
  - Look for local ways to gather information
    - Trivial to make desirable exact potential game out of coordination games
    - Possible to use other forms, may require modifying observations or defining specific network processes
- Broadly, if you have an analytic model with desirable properties, design your cognitive radio network to conform to that model

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Extra Slides

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Points to Remember

- In addition to interactive decisions, timing and distribution of information are critical
- Policies are a good way to limit worst case scenarios
- Additive cost functions can shape behavior
- Collaboration and centralization can eliminate interactive decision problems
- Punishment can limit incentives to cheat on collaborative agreements
    - But is very sensitive to the design
- Under special conditions (bilateral symmetric interference), interactive decisions form a virtuous cycle

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Designing Cognitive Radio Networks to Yield Desired Behavior

Policy, Cost Functions, Global Altruism, Potential Games

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Potential Problems with Networked Cognitive Radios

Distributed

- Infinite recursions
- Instability (chaos)
- Vicious cycles ——— Decision Interaction
- Adaptation collisions ——— Timing
- Equitable distribution of resources
- Byzantine failure
- Information distribution ——— Distribution of Trusted Accurate Information

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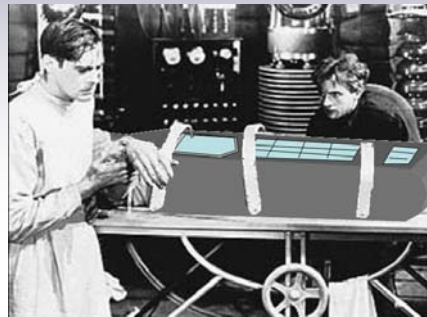Ph: (540) 230-6012
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Working with Interactive Decisions

- Design network to be a potential game
  - Any self interested decision process will converge
- Limit decisions to processes known to converge
  - Best responses in a supermodular game
- Limit effects of interactions
  - Policy
- Eliminate interaction
  - Centralize decision making
  - Collaboration
  - Repeated game with punishment

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Policy

- Concept: Constrain the available actions so the worst cases of distributed decision making can be avoided
- Not a new concept –
  - Policy has been used since there's been an FCC
- What's new is assuming decision makers are the radios instead of the people controlling the radios
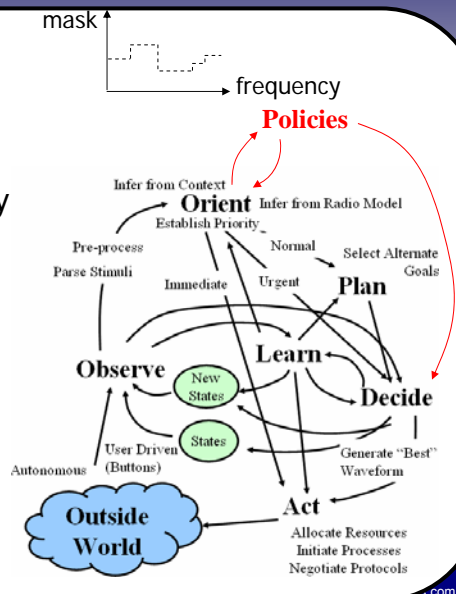
Γ	N
n	$(9.6, 9.6)$

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Policy applied to radios instead of humans

mask

frequency

Policies

- Need a language to convey policy
  - Learn what it is
  - Expand upon policy later
- How do radios interpret policy
  - Policy engine?
- Need an enforcement mechanism
  - Might need to tie in to humans
- Need a source for policy
  - Who sets it?
  - Who resolves disputes?
- Logical extreme can be quite complex, but logical extreme may not be necessary.

Infer from Context
Orient Infer from Radio Model
Establish Priority

Pre-process
Parse Stimuli   Normal   Select Alternate
                                     Goals
        Immediate   Urgent   **Plan**

Learn

Observe   New
              States

                                Decide

User Driven   States   Generate "Best"
(Buttons)   Waveform
Autonomous

Outside World

Act
Allocate Resources
Initiate Processes
Negotiate Protocols

802.22 Example Policies

- Detection
  - Digital TV: -116 dBm over a 6 MHz channel
  - Analog TV: -94 dBm at the peak of the NTSC (National Television System Committee) picture carrier
  - Wireless microphone: -107 dBm in a 200 kHz bandwidth.
- Transmitted Signal
  - 4 W **E**ffective **I**sotropic **R**adiated **P**ower (**EIRP**)
  - Specific spectral masks

C. Cordeiro, L. Challapali, D. Birru, S. Shankar, "IEEE 802.22: The First Worldwide Wireless Standard based on Cognitive Radios," *IEEE DySPAN2005*, Nov 8-11, 2005 Baltimore, MD.
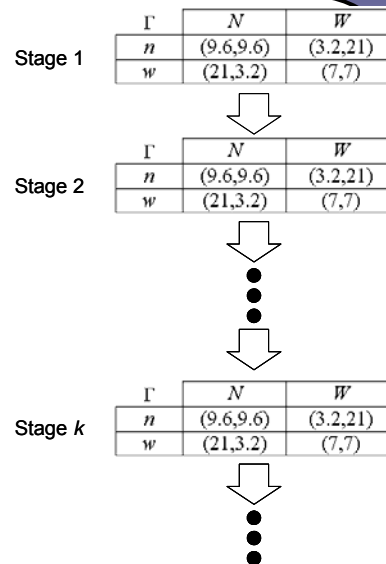
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Repeated Games

- **Same game is** *repeated*
  - Indefinitely
  - Finitely
- **Players consider discounted payoffs across multiple stages**
  - Stage k $\tilde{u}_i(a^k) = \delta^k u_i(a^k)$
  - Expected value over all future stages

$$\widehat{u}_i((a^k)) = \sum_{k=0}^{\infty} \delta^k u_i(a^k)$$

Γ	N	W
n	(9.6,9.6)	(3.2,21)
w	(21,3.2)	(7,7)

Stage 1

Γ	N	W
n	(9.6,9.6)	(3.2,21)
w	(21,3.2)	(7,7)

Stage 2

Γ	N	W
n	(9.6,9.6)	(3.2,21)
w	(21,3.2)	(7,7)

Stage k

Impact of Strategies

- Rather than merely reacting to the state of the network, radios can choose their actions to influence the actions of other radios
- Threaten to act in a way that minimizes another radio's performance unless it implements the desired actions
- Common strategies
  - Tit-for-tat
  - Grim trigger
  - Generous tit-for-tat
- Play can be forced to any "feasible" payoff vector with proper selection of punishment strategy.

Theorem 4.5: Grim Trigger Folk theorem [Fudenberg_91]
In a repeated game with an infinite horizon and discounting, for every feasible payoff vector $v > \underline{v_i}$ for all $i \in N$, there exists a $\underline{\delta} < 1$ such that for all $\delta \in (\underline{\delta}, 1)$ there is a steady-state with payoffs v.


Impact of Communication on Strategies

- Players agree to play in a certain manner
- Threats can force play to almost any state
  - Breaks down for finite number of stages

|	Nada	C	N
nada	0,0	-5,5	-100,0
c	5,-5	-1,1	-100,-1
n	0,-100	-1,-100	-100,-100

Improvement from Punishment

- Throughput/unit power gains be enforcing a common received power level at a base station
- Punishment by jamming
- Without benefit to deviating, players can operate at lower power level and achieve same throughput
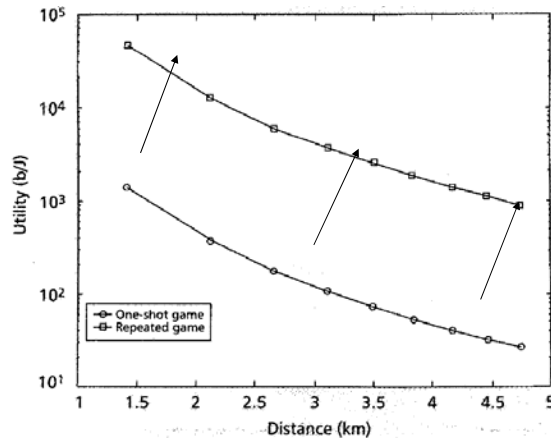
A. MacKenzie and S. Wicker, "Game Theory in Communications: Motivation, Explanation, and Application to Power Control," *Globecom2001*, pp. 821-825.

Instability in Punishment

- Issues arise when radios aren't directly observing actions and are punishing with their actions without announcing punishment
- Eventually, a deviation will be falsely detected, punished and without signaling, this leads to a cascade of problems
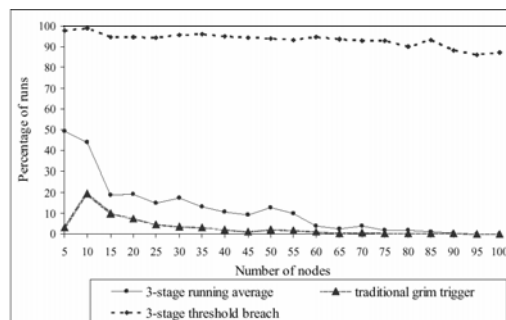
V. Srivastava, L. DaSilva, "Equilibria for Node Participation in Ad Hoc Networks – An Imperfect Monitoring Approach," ICC 06, June 2006, vol 8, pp. 3850-3855

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Comments on Punishment

- Works best with a common controller to announce
- Problems in fully distributed system
  - Need to elect a controller
  - Otherwise competing punishments, without knowing other players' utilities can spiral out of control
- Problems when actions cannot be directly observed
  - Leads to Byzantine problem
- No single best strategy exists
  - Strategy flexibility is important
  - Significant problems with jammers (they nominally receive higher utility when "punished"
- Generally better to implement centralized controller
  - Operating point has to be announced anyways

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Cost Adjustments

- Concept: Centralized unit dynamically adjusts costs in radios' objective functions to ensure radios operate on desired point

$$\tilde{u}_i(a) = u_i(a) + c_i(a)$$

- Example: Add -12 to use of wideband waveform

Γ	N	W
n	(9.6,9.6)	(3.2,9)
w	(9,3.2)	(-5,-5)

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Comments on Cost Adjustments

- Permits more flexibility than strict rules
  - If a radio really needs to deviate, then it can
- Easy to turn off and on as a policy tool
  - Example: protected user shows up in a channel, cost to use that channel goes up
  - Example: prioritized user requests channel, other users' cost to use prioritized user's channel goes up (down if when done)

Potential Games

The best game model for designing cognitive radio networks since…..

Sliced bread

Potential Games

- Existence of a function (called the potential function, V), that reflects the change in utility seen by a unilaterally deviating player.
- Cognitive radio interpretation:
  - Every time a cognitive radio unilaterally adapts in a way that furthers its own goal, some real-valued function increases.

Potential Game	Relationship ($\forall i \in N, \forall a \in A$)
Exact (EPG)	$u_i(b_i, a_{-i}) - u_i(a_i, a_{-i}) = V(b_i, a_{-i}) - V(a_i, a_{-i})$
Weighted (WPG)	$u_i(b_i, a_{-i}) - u_i(a_i, a_{-i}) = \alpha_i \left[V(b_i, a_{-i}) - V(a_i, a_{-i}) \right]$
Ordinal (OPG)	$u_i(b_i, a_{-i}) - u_i(a_i, a_{-i}) > 0 \Leftrightarrow V(b_i, a_{-i}) - V(a_i, a_{-i}) > 0$
Generalized Ordinal (GOPG)	$u_i(b_i, a_{-i}) - u_i(a_i, a_{-i}) > 0 \Rightarrow V(b_i, a_{-i}) - V(a_i, a_{-i}) > 0$
Generalized ε (GεPG)	$u_i(b_i, a_{-i}) > u_i(a_i, a_{-i}) + \varepsilon_1 \Rightarrow V(b_i, a_{-i}) > V(a_i, a_{-i}) + \varepsilon_2$

Exact Potential Game Forms

- Many exact potential games can be recognized by the form of the utility function

Game	Utility Function Form	Potential Function
Coordination Game	$u_i(a) = C(a)$	$V(a) = C(a)$
Dummy Game	$u_i(a) = D_i(a_{-i})$	$V(a) = c, c \in \mathbb{R}$
Coordination-Dummy Game	$u_i(a) = C(a) + D_i(a_{-i})$	$V(a) = C(a)$
Self-Motivated Game	$u_i(a) = S_i(a_i)$	$V(a) = \sum_{i \in N} S_i(a_i)$
Bilateral Symmetric Interaction (BSI) Game	$u_i(a) = \sum_{j \in M \setminus \{i\}} w_{ij}(a_i, a_j) - S_i(a_i)$ where $w_{ij}(a_i, a_j) = w_{ji}(a_j, a_i)$	$V(a) = \sum_{i \in N} \sum_{j=1}^{i-1} w_{ij}(a_i, a_j) - \sum_{i \in N} S_i(a_i)$
Multilateral Symmetric Interaction (MSI) Game	$u_i(a) = \sum_{\{S \in 2^N : i \in S\}} w_{S,i}(a_S) + D_i(a_{-i})$ where $w_{S,i}(a_S) = w_{S,j}(a_S) \forall i, j \in S$	$V(a) = \sum_{S \in 2^N} w_S(a_S)$

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Implications of Monotonicity

- Monotonicity implies
  - Existence of steady-states (maximizers of V)
  - Convergence to maximizers of V for numerous combinations of decision timings decision rules – all self-interested adaptations
- Does not mean that that we get good performance
  - Only if V is a function we want to maximize

Decision Rules	Timings			
	Round-Robin	Random	Synchronous	Asynchronous
Best Response	1,2,4	1,2,4	-	1,2
Exhaustive Better Response	1,2	1,2	-	1,2
Random Better Response[a]	1,2,4	1,2,4	1,2	1,2
Random Better Response[b]	1,2	1,2	-	1,2
ε-Better Response[c]	1,2,3,4	1,2,3,4	-	1,2,3
Intelligently Random Better Response	1,4	1,4	-	1,2
Directional Better Response[c]	4	4	-	-
Averaged Best Response[d]	3,4	3,4	-	-

(a) Definition 4.12, (b) Definition 4.13, (c) Convergence to an ε-NE, (d) u_i quasi-concave in a_i
1. Finite game, 2. Infinite game with FIP, 3. Infinite game with AFIP, 4. Infinite game with bounded continuous potential function (implication of D^V)
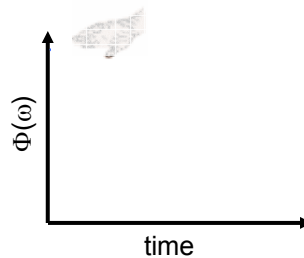

Other Potential Game Properties

- All finite potential games have FIP
- All finite games with FIP are potential games
  - Very important for ensuring convergence of distributed cognitive radio networks
- -V is a is a Lyapunov function for isolated maximizers
- Stable NE solvable by maximizers of V
- Linear combination of exact potential games is an exact potential game
- Maximizer of potential game need not maximize your objective function
  - Cognitive Radios' Dilemma is a potential game

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Interference Reducing Networks

- Concept
  - Cognitive radio network is a potential game with a potential function that is negation of observed network interference
- Definition
  - A network of cognitive radios where each adaptation decreases the sum of each radio's observed interference is an *IRN*

$$\Phi(\omega) = \sum_{i \in N} I_i(\omega)$$

- Implementation:
  - Design algorithms such that network is a potential game with $\Phi \propto -V$
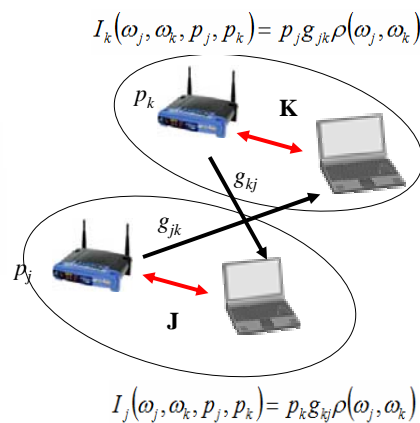

Designing An IRN

- Link Bilateral Symmetric Interference (BSI) if

$$I_j(\omega_j, \omega_k, p_j, p_k) = I_k(\omega_j, \omega_k, p_j, p_k)$$
$$g_{jk} p_j \rho(\omega_j, \omega_k) = g_{kj} p_k \rho(\omega_k, \omega_j)$$
$$\forall \omega_j \in \Omega_j, \forall \omega_k \in \Omega_k$$

- Network BSI if link BSI holds for the observation metrics of all pairs of decision processes

$$I_k(\omega_j, \omega_k, p_j, p_k) = p_j g_{jk} \rho(\omega_j, \omega_k)$$

$$I_j(\omega_j, \omega_k, p_j, p_k) = p_k g_{kj} \rho(\omega_j, \omega_k)$$

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Selfish adaptations reduce sum network interference when BSI holds

- Sum network interference

$$\Phi(\omega, p) = \sum_{j \in N} I_j(\omega, p)$$

- With two links and BSI

$$I_j\left(\omega_j^1, \omega_k, p\right) < I_j\left(\omega_j^2, \omega_k, p\right) \Rightarrow I_k\left(\omega_j^1, \omega_k, p\right) < I_k\left(\omega_j^2, \omega_k, p\right)$$

$$\Phi\left(\omega_j^1, \omega_k, p\right) < \Phi\left(\omega_j^2, \omega_k, p\right)$$

$$\Phi\left(\omega_j^2, \omega_k, p\right) - \Phi\left(\omega_j^1, \omega_k, p\right) = 2\left(I_j\left(\omega_j^2, \omega_k, p\right) - I_j\left(\omega_j^1, \omega_k, p\right)\right)$$

Network sees twice the benefit of the selfish adapter

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With multiple links and BSI, the same relationship is seen

$$I_j(\omega, p) = \sum_{k \in N \setminus j} p_k g_{kj} \rho(\omega_j, \omega_k)$$

Change in interference for selfish adapter

$$I_j\left(\omega_j^1, \omega_{-j}, p\right) - I_j\left(\omega_j^2, \omega_{-j}, p\right) = \sum_{k \in N \setminus j} p_k g_{kj} \rho\left(\omega_j^1, \omega_{-j}\right) - \sum_{k \in N \setminus j} p_k g_{kj} \rho\left(\omega_j^2, \omega_{-j}\right)$$

Interference Terms Not Influenced by j

$$\Phi\left(\omega_j^1, \omega_{-j}, p\right) - \Phi\left(\omega_j^2, \omega_{-j}, p\right) = \overbrace{\sum_{k \in N \setminus j} \sum_{m \in N \setminus \{j,k\}} p_k g_{km} \rho(\omega_k, \omega_m) - \sum_{k \in N \setminus j} \sum_{m \in N \setminus \{j,k\}} p_k g_{km} \rho(\omega_k, \omega_m) + \cdots}$$

$$\sum_{k \in N \setminus j} p_k g_{kj} \rho\left(\omega_j^1, \omega_k\right) - \sum_{k \in N \setminus j} p_k g_{kj} \rho\left(\omega_j^2, \omega_k\right) + \cdots \quad \text{Interference Seen by } j$$

$$\sum_{k \in N \setminus j} p_j g_{jk} \rho\left(\omega_j^1, \omega_k\right) - \sum_{k \in N \setminus j} p_j g_{jk} \rho\left(\omega_j^2, \omega_k\right) \quad \text{Interference Caused by } j$$

$$\Phi\left(\omega_j^1, \omega_{-j}, p\right) - \Phi\left(\omega_j^2, \omega_{-j}, p\right) = 2\left(I_j\left(\omega_j^1, \omega_{-j}, p\right) - I_j\left(\omega_j^2, \omega_{-j}, p\right)\right)$$

Network sees twice the benefit of the selfish adapter

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This correlation between selfish and social benefit yields desirable behavior

- Convergence
  - ***ALL*** sequences of unilateral selfish adaptations induce monotonically decreasing network interference levels
  - For finite waveform sets, completely unsynchronized adaptations form absorbing Markov chains
- Optimality of steady-states
  - Assuming exhaustive adaptations, interference minimizers are the only steady-states
- Stability
  - Sum network interference is a Lyapunov function in neighborhoods of isolated interference minimizers
  - In practice, many minimizers aren't isolated, so some hysteresis is needed
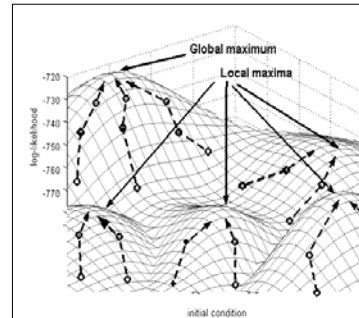

Figure from Fig 2.6 in I. Akbar, "Statistical Analysis of Wireless Systems Using Markov Models," PhD Dissertation, Virginia Tech, January 2007

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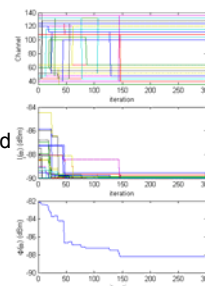147 Mill Ridge Rd, Ste 119
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This connection can be used to achieve minimal complexity

- Because selfish behavior is good for the network, no need to directly consider impact on other links
  - Means virtually no bandwidth lost to control messages
- Because selfish behavior is based solely on its own observations, there's no need to burden the network distributing observations
- Because unsynchronized adaptations converge, there no need for clock distribution
  - Will converge faster if properly synchronized
- Because *ALL* selfish adaptations converge, even trial and error, decision rules can be very simple
  - As simple as search through weighted RSSI measurements
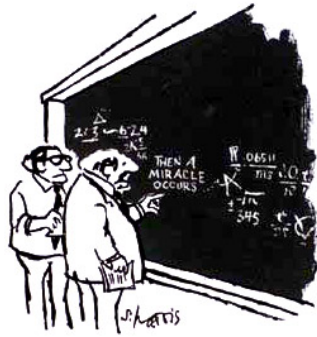
Example unsynchronized behavior

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Situations where BSI occurs

$$g_{kj}p_k\rho\left(\omega_j,\omega_k\right)=g_{jk}p_j\rho\left(\omega_k,\omega_j\right)$$

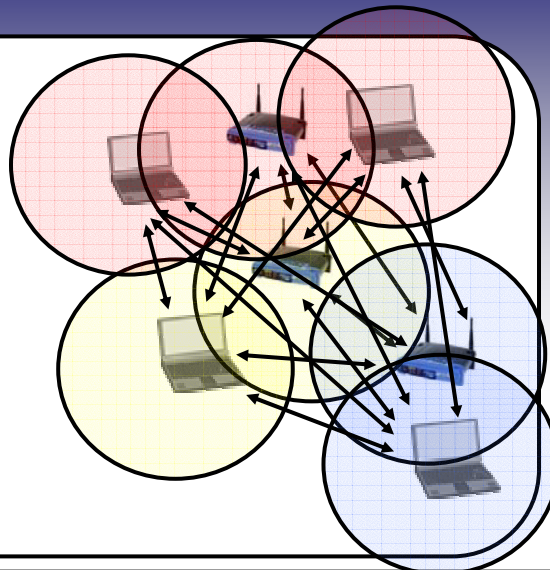"I THINK YOU SHOULD BE MORE EXPLICIT HERE IN STEP TWO."

- Isolated Network Clusters
  - All devices communicate with a common access node with identical received powers.
  - Clusters are isolated in signal space
- Close Proximity Networks
  - All devices are sufficiently close that waveform correlation effects dominate
- Controlled Observation Processes
  - Leverage knowledge of waveform protocol to create observation metrics which achieve BSI for the allowed adaptations

---

BSI by Subtraction

- Huge number of interference sources in a network
- Concept: constrain observations to only consider symmetric interference sources

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An IRN 802.11 DFS Algorithm

- Suppose each **access node measures** the received signal **power** and **frequency** of the RTS/CTS (or BSSID) messages sent by observable access nodes in the network
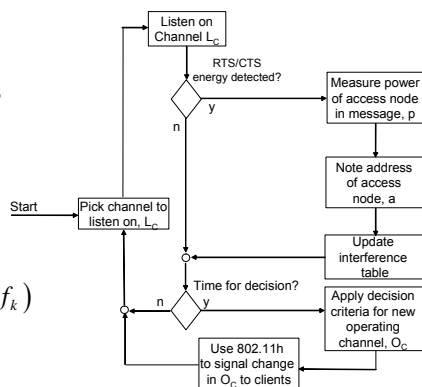 - Ignore client interference
- Assumed out-of-channel interference is negligible and RTS/CTS transmitted at same power

$$u_i(f) = -I_i(f) = -\sum_{k \in N \setminus i} g_{ki} p_k \sigma(f_i, f_k)$$

$$\sigma(f_i, f_k) = \begin{cases} 1 & f_i = f_k \\ 0 & f_i \neq f_k \end{cases}$$

$$g_{jk} p_j \sigma(f_j, f_k) = g_{kj} p_k \sigma(f_k, f_j)$$

Start → Pick channel to listen on, L_C

Listen on Channel L_C

RTS/CTS energy detected?
- n
- y → Measure power of access node in message, p

Note address of access node, a

Update interference table

Time for decision?
- n
- y → Apply decision criteria for new operating channel, O_C

Use 802.11h to signal change in O_C to clients

J. Neel, J. Reed, "Performance of Distributed Dynamic Frequency Selection Schemes for Interference Reducing Networks," *Milcom 2006*, Washington DC, October 23-25, 2006
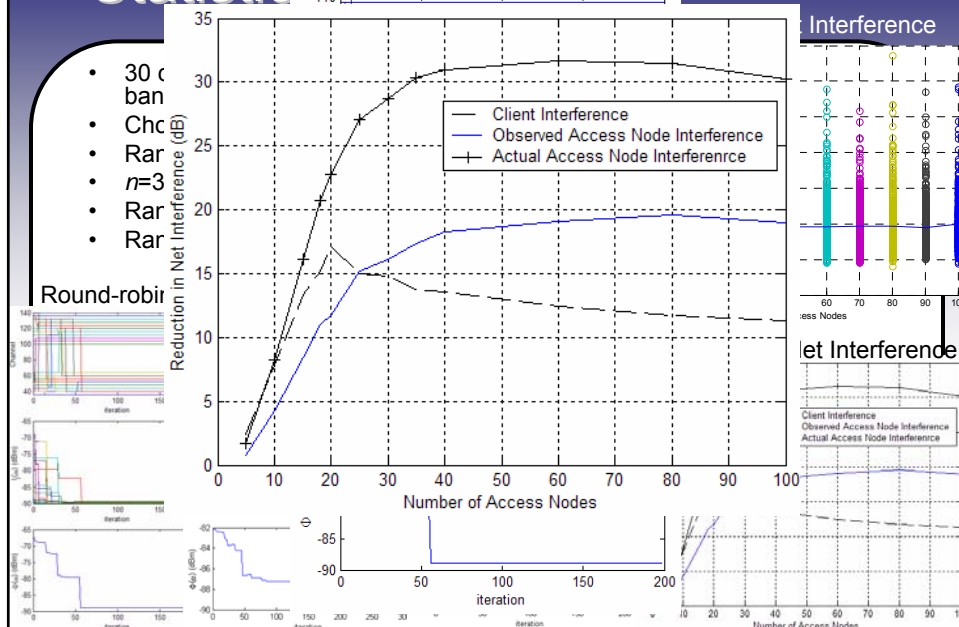
Statistics

- 30 c... ban...
- Cho...
- Ran...
- *n*=3...
- Ran...
- Ran...

Round-robin

Client Interference
Observed Access Node Interference
Actual Access Node Interferenrce

Reduction in Net Interference (dB)

Number of Access Nodes

BSI by Addition (Ad-hoc)

- Define players (decision processes) as links
  - Both sides of a link collaborate to make a decision
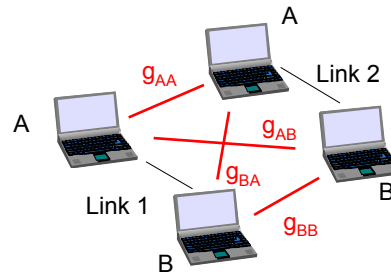 - Permits incorporation of observations from both radios

- Consider Interference levels
  - Link 1
    - A $\quad p(g_{AA} + g_{AB})\rho(f_1, f_2)$
    - B $\quad p(g_{BA} + g_{BB})\rho(f_1, f_2)$
    - A+B $\quad p(g_{AA} + g_{AB} + g_{BA} + g_{BB})\rho(f_1, f_2)$
  - Link 2
    - A $\quad p(g_{AA} + g_{BA})\rho(f_1, f_2)$
    - B $\quad p(g_{AB} + g_{BB})\rho(f_1, f_2)$
   - A+B $\quad p(g_{AA} + g_{AB} + g_{BA} + g_{BB})\rho(f_1, f_2)$

- Concept: Assuming TDD and equal powers, device-to-device interference is BSI. Observations formed as sum of device measurements for a link or a cluster is then also BSI.

A
g_{AA} Link 2
A g_{AB}
g_{BA} B
Link 1 g_{BB}
B

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Aggregate Statistics for P2P Network

Reduce interference by 30 dB

Support 16 x more links
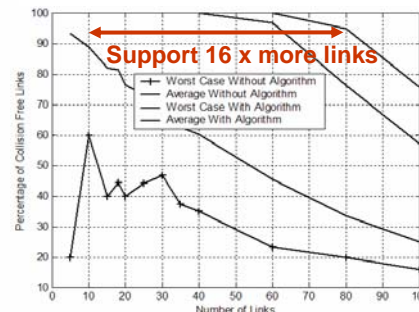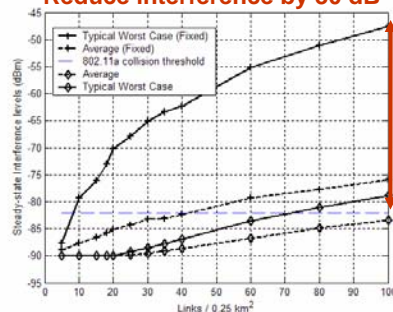
- Similar algorithm but cognitive decision processes span links
- No coordination/messaging between decision processes
- Localized reasoning yields steady-state performance equivalent to centralized local search
- No need to recover interfering signal – interference range is detection range

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BSI by Multiplication 1

- Power control + channel allocation can yield much better performance, but power control violates BSI assumptions

$$g_{jk} p_j \rho\left(\omega_j, \omega_k\right) = g_{kj} p_k \rho\left(\omega_k, \omega_j\right) \ ??$$

- Solution: devices weight interference observations by own power level

$$\tilde{I}_j(\omega) = p_j I_j(\omega) = p_j \sum_{k \in N \setminus j} g_{kj} p_k \rho\left(\omega_j, \omega_k\right)$$

$$g_{jk} p_j p_k \rho\left(\omega_j, \omega_k\right) = g_{kj} p_k p_j \rho\left(\omega_k, \omega_j\right)$$
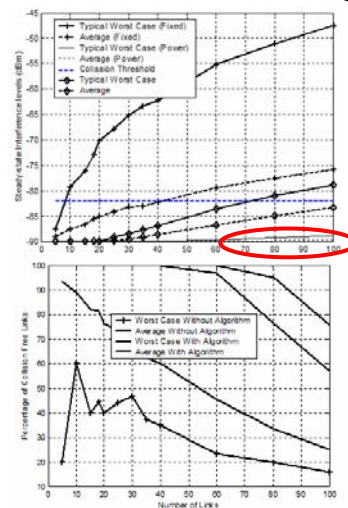
- Comments
  - Some interaction between power and channel choices
  - Should not be used as objective for setting power levels

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Aggregate Statistics for P2P Network

- Power control to achieve 16 dB SINR reception (typical SNR needed to recover 64-QAM with BER of 10^{-5})

- Lower slope & much less interference

- At 400 links/km² network is actually operating collision free (worst case interference remains below collision threshold)

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BSI by Multiplication 2

- Frequently, we want to prioritize access of certain transmissions
  - Voice versus email
  - General vs private

$$\tilde{I}_j(\omega) = w_j \sum_{k \in N \setminus j} w_k g_{kj} p_k \rho(\omega_j, \omega_k)$$

- Can accommodate this goal while preserving BSI by multiplying interference observations by weights of detected signals and then weighting aggregate levels by own weight

$$w_j w_k g_{jk} p \rho(\omega_j, \omega_k) = w_j w_k g_{kj} p \rho(\omega_k, \omega_j)$$

- Comments
  - Need some mechanism for distributing weighting factors
  - Interference range != detection range because of need to recover signal characteristics

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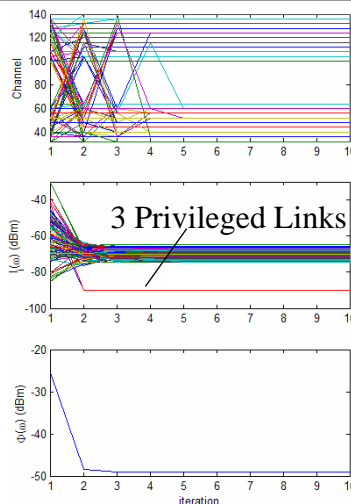
Example Simulation

- Basic parameters
  - 100 randomly distributed links in 0.5 x 0.5 km area
  - Round-robin iterations
  - 3 privileged links weighted at factor 100, others at 1
- 3 privileged links get their own channels
- (Weighted) Sum interference retains monotonic characteristic
- Note faster convergence from coordinated timings

3 Privileged Links

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BSI Design Summary

- BSI is a conceptually simple concept to evaluate

$$g_{jk} p_j \rho\left(\omega_j, \omega_k\right) = g_{kj} p_k \rho\left(\omega_k, \omega_j\right)$$

$$\forall \omega_j \in \Omega_j, \forall \omega_k \in \Omega_k$$

- When BSI holds
  - Network self-optimizes from selfish adaptations
    - No need to coordinate
    - No need to centralize
  - Complexity / overhead can be made very low
- BSI does not naturally occur frequently, but can be synthesized by careful design of the observation/objective functions
- CRT has developed techniques for synthesizing BSI observations for
  - Frequency, time, power, MIMO, beam forming, OFDM systems, accounting for varying traffic intensities, varying user priorities
 - Combinations of the preceding
- Applicability to
  - Ad-hoc nets, uncoordinated access points (e.g., apartments), femto-cells, home gateways, sensor nets
  - 802.11a/b/g/n, WiMAX, 802.22
  - Biggest benefit is in rapidly changing environments, large networks, and networks where management is impractical

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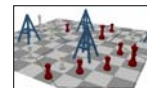147 Mill Ridge Rd, Ste 119
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Presentation Takeaways

- Insufficient to only consider cognitive radio adaptations effect over a single link
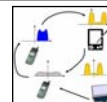
- Game theory provides nice tools for modeling and analyzing this interactive decision problem

- The bilateral symmetric interference condition permit the use of low complexity, "zero-overhead" algorithms to minimize network interference

- Powerful algorithms (30 dB) can be designed by focusing on the radios' objectives and observations
  - Greed is good. (at least when BSI holds)

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Items to Remember

- In addition to interactive decisions, timing and distribution of information are critical
- Policies are a good way to limit worst case scenarios
- Additive cost functions can shape behavior
- Collaboration and centralization can eliminate interactive decision problems
- Punishment can limit incentives to cheat on collaborative agreements
  - But is very sensitive to the design
- Under special conditions (bilateral symmetric interference), interactive decisions form a virtuous cycle

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Analysis Models

Model (Section number)	Basic model	Identification
Dynamical Systems (3.1)	evolution equation $a\left(t^{k+1}\right)=d^{t}\left(a\left(t^{k}\right)\right)$	Assumed to exist.
Contraction Mappings (3.2)	$\left\|d(a)-d(b)\right\|\leq\alpha\left\|a-b\right\|$ $\forall b,a\in A$	Apply definition.
Standard Interference Function Power Control (3.2.4.1)	$d_{j}\left(\mathbf{p}\left(t^{k}\right)\right)=p_{j}\left(t^{k}\right)I_{j}\left(\mathbf{p}\left(t^{k}\right)\right)$	$I(\mathbf{p})$ satisfies positivity, montonicity, and scalability
Ergodic Markov Chain (3.3.2)	$P\left(a\left(t^{k+1}\right)=a^{k}\mid a(0),\ldots,a(t)\right)$ $=P\left(a\left(t^{k+1}\right)=a^{k}\mid a\left(t^{k}\right)\right)$	$\exists\,k$ such that $\mathbf{P}^{k}$ has all positive entries
Absorbing Markov Chain (3.3.3)	$\mathbf{P}'=\left[\begin{array}{c\|c} \mathbf{Q} & \mathbf{R} \\ \hline \mathbf{0} & \mathbf{I}^{ab} \end{array}\right]$	Apply model definition

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Model Steady States

Model (Section number)	Existence	Identification
Dynamical Systems (3.1)	Maybe, evaluate Leray-Schauder-Tychonoff theorem on evolution equation	Exhaustive Search, Solve $d\left(a^*\right) = a^*$
Contraction Mappings (3.2)	Yes (Banach's Theorem)	Recursion (Unique steady-state)
Standard Interference Function Power Control (3.2.4.1)	Yes ([Yates_95])	Recursion (Unique steady-state), $\mathbf{Z}\mathbf{p} = \vec{\gamma}$
Ergodic Markov Chain (3.3.2)	Yes (Ergodocity theorem)	Recursion (Unique distribution), Solve $\boldsymbol{\pi}^{*T}\mathbf{P} = \boldsymbol{\pi}^{*T}$
Absorbing Markov Chain (3.3.3)	Yes (Definition)	$p_{mm} = 1$

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Model Convergence

Model (Section number)	Sensitivity	Rate
Dynamical Systems (3.1)	Apply Lyapunov's direct method (when possible)	No general technique
Contraction Mappings (3.2)	Everywhere convergent	$\left\| a\left(t^k\right), a^* \right\| \leq \dfrac{\alpha^k}{1-\alpha}\left\| a\left(t^1\right), a\left(t^0\right) \right\|$
Standard Interference Function Power Control (3.2.4.1)	Everywhere convergent	$\left\| \mathbf{p}\left(t^k\right), \mathbf{p}^* \right\| \leq \alpha^k \left\| \mathbf{p}(0), \mathbf{p}^* \right\|$ $\alpha = \rho(\mathbf{H})$
Ergodic Markov Chain (3.3.2)	Converges to distribution from all starting distributions	Transition matrix dependent
Absorbing Markov Chain (3.3.3)	$\mathbf{B} = \mathbf{NR}$	$\mathbf{t} = \mathbf{N}\mathbf{1}$

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Model Stability

Model (Section number)	Lyapunov Stability	Attractivity
Dynamical Systems (3.1)	Apply Lyapunov's direct method (when possible)	Apply Lyapunov's direct method (when possible)
Contraction Mappings (3.2)	Global	Global
Standard Interference Function Power Control (3.2.4.1)	Global	Global
Ergodic Markov Chain (3.3.2)	No	No
Absorbing Markov Chain (3.3.3)	Not guaranteed.	If unique absorbing state