Presentation Overview

- Overview of Cognitive Radio
- Interactive Decision Problem
- A “Quick” Review of Game Theory
- Designing Cognitive Radio Networks
- Examples of Networked Cognitive Radios
- Future Directions in Cognitive Radio

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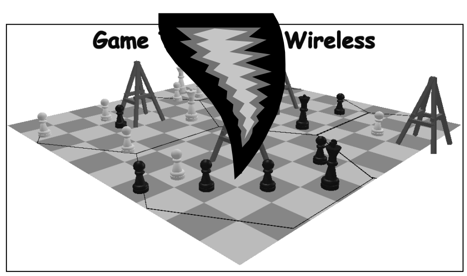
A Whirlwind Review of Game Theory

Normal form games, Nash equilibria, Pareto efficiency, Improvement Paths, Noise



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Game

1. A (well-defined) set of 2 or more players
2. A set of actions for each player.
3. A set of preference relationships for each player for each possible action tuple.

- More elaborate games exist with more components but these three must always be there.
- Some also introduce an outcome function which maps action tuples to outcomes which are then valued by the preference relations.
- Games with just these three components (or a variation on the preference relationships) are said to be in **Normal** form or **Strategic** Form

Set of Players (decision makers)

- N – set of n players consisting of players "named" {1, 2, 3,…,i, j,…,n}
- Note the n does not mean that there are 14 players in every game.
- Other components of the game that "belong" to a particular player are normally indicated by a subscript.
- Generic players are most commonly written as i or j.
- Usage: N is the SET of players, n is the number of players.
- $N \setminus i = \{1,2,\ldots,i-1, i+1 ,\ldots, n\}$ All players in N except for i

Actions

A_i – Set of available actions for player i

a_i – A particular action chosen by i, $a_i \in A_i$

A – Action Space, Cartesian product of all A_i

$A = A_1 \times A_2 \times \cdots \times A_n$

a – Action tuple – a point in the Action Space

A_{-i} – Another action space A formed from

$A_{-i} = A_1 \times A_2 \times \cdots \times A_{i-1} \times A_{i+1} \times \cdots \times A_n$

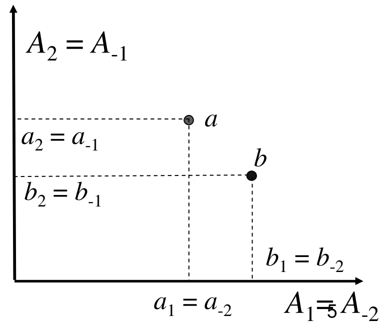
a_{-i} – A point from the space A_{-i}

$A = A_i \times A_{-i}$

Example Two Player Action Space

$A_1 = A_2 = [0\ \infty)$

$A = A_1 \times A_2$



Preference Relations (1/2)

Preference Relation expresses an individual player's desirability of one outcome over another (A binary relationship)

$\succsim_i$ Preference Relationship (prefers at least as much as)

$o \succsim_i o^*$ o is preferred at least as much as o^* by player i

$\succ_i$ Strict Preference Relationship (prefers strictly more than)

$o \succ_i o^*$ iff $o \succsim_i o^*$ but not $o^* \succsim_i o$

$\sim_i$ "Indifference" Relationship (prefers equally)

$o \sim_i o^*$ iff $o \succsim_i o^*$ and $o^* \succsim_i o$

Preference Relationship (2/2)

- Games generally assume the relationship between actions and outcomes is invertible so preferences can be expressed over action vectors.
- Preferences are really an *ordinal* relationship
  - Know that player prefers one outcome to another, but quantifying by how much introduces difficulties

Utility Functions (1/2) (Objective Fcns, Payoff Fcns)

A mathematical description of preference relationships.

Maps action space to set of real numbers.

$$u_i : A \rightarrow \mathbb{R}$$

Preference Relation then defined as

$$a \succsim_i a^* \text{ iff } u_i(a) \geq u_i(a^*)$$

$$a \succ_i a^* \text{ iff } u_i(a) > u_i(a^*)$$

$$a \sim_i a^* \text{ iff } u_i(a) = u_i(a^*)$$

Utility Functions (2/2)

By quantifying preference relationships all sorts of valuable mathematical operations can be introduced.

Also note that the quantification operation is not unique as long as relationships are preserved. Many map preference relationships to [0,1].

Example

Jack prefers Apples to Oranges

$$Apples \succ_{Jack} Oranges \Leftrightarrow u_{Jack}(Apples) > u_{Jack}(Oranges)$$

$a)\ u_{Jack}(Apples) = 1,\ u_{Jack}(Oranges) = 0$

$b)\ u_{Jack}(Apples) = -1,\ u_{Jack}(Oranges) = -7.5$

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Variety of game models

- Normal Form Game $<N,A,\{u_i\}>$
  - Synchronous play
  - *T* is a singleton
  - Perfect knowledge of action space, other players' goals (called utility functions)
- Repeated Game $<N,A,\{u_i\},\{d_i\}>$
  - Repeated synchronous play of a normal form game
  - *T* may be finite or infinite
  - Perfect knowledge of action space, other players' goals (called utility functions)
  - Players may consider actions in future stages and current stages
    - Strategies (modified d_i)
- Asynchronous myopic repeated game $<N,A,\{u_i\},\{d_i\},T>$
  - Repeated play of a normal form game under various timings
  - Radios react to most recent stage, decision rule is "intelligent"
- Many others in the literature and in the dissertation

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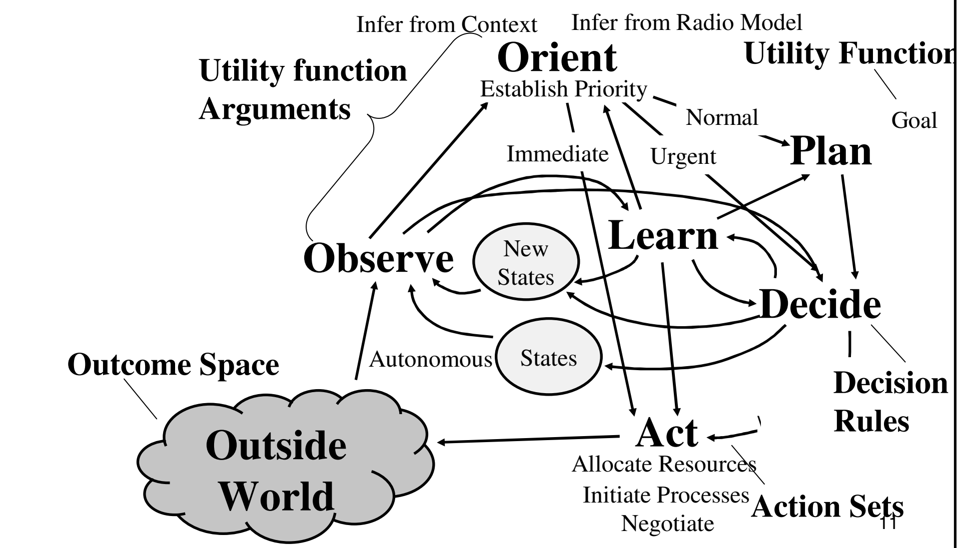
Cognitive radios are naturally modeled as players in a game



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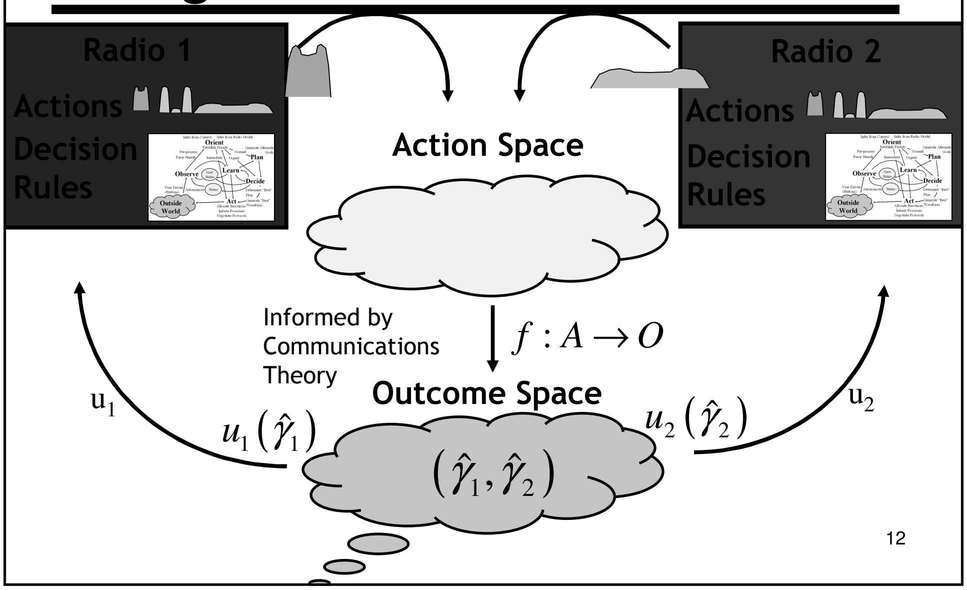
Adapted From Mitola, "Cognitive Radio for Flexible Mobile Multimedia Communications ", IEEE Mobile Multimedia Conference, 1999, pp 3-10.

Interaction is naturally modeled as a game



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Some differences between game models and cognitive radio network model

- Assuming numerous iterations, normal form game only has a single stage.
  - Useful for compactly capturing modeling components at a single stage
  - Normal form game properties will be exploited in the analysis of other games

	Player	Cognitive Radio
Knowledge	Knows *A*	Can learn *O* (may know or learn *A*)
$f: A \rightarrow O$	Invertible Constant Known	Not invertible (noise) May change over time (though relatively fixed for short periods) Has to learn
Preferences	Ordinal	Cardinal (goals)

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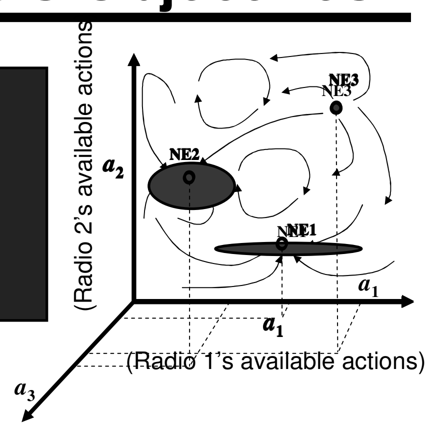
Network Analysis Objectives

focus

1. Steady state characterization
2. Steady state optimality
3. Convergence
4. Stability/Noise
5. Scalability



Steady State Characterization

How do initial conditions impact the system steady state?
Do these outcomes maximize the system target parameters?
Do previously optimal steady states remain optimal?

Steady-states

- Recall model of <N,A,{d_i},T> which we characterize with the evolution function d
- Steady-state is a point where $a^* = d(a^*)$ for all $t \geq t^*$
- Obvious solution: solve for fixed points of d.
- For non-cooperative radios, if a^* is a fixed point under synchronous timing, then it is under the other three timings.
- Works well for convex action spaces
  - Not always guaranteed to exist
  - Value of fixed point theorems
- Not so well for finite spaces
  - Generally requires exhaustive search

Nash Equilibrium

"A steady-state where each player holds a correct expectation of the other players' behavior and acts rationally." - Osborne
An action vector from which no player can profitably unilaterally deviate.

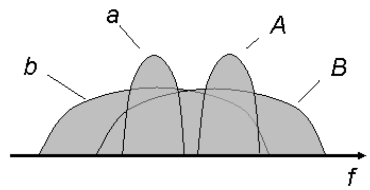
Definition

An action tuple a is a NE if for every $i \in N$ $u_i(a_i, a_{-i}) \geq u_i(b_i, a_{-i})$ for all $b_i \in A_i$.

Note showing that a point is a NE says nothing about the process by which the steady state is reached. Nor anything about its uniqueness.
Also note that we are implicitly assuming that only pure strategies are possible in this case.

Examples

- Cognitive Radios' Dilemma
  - Two radios have two signals to choose between {*n*,*w*} and {*N*,*W*}
  - *n* and *N* do not overlap
  - Higher throughput from operating as a high power wideband signal when other is narrowband
- Jamming Avoidance
  - Two channels
  - No NE



Γ	N	W
n	(9.6,9.6)	(3.2,21)
w	(21,3.2)	(7,7)

Jammer

Transmitter		0	1
	0	(-1,1)	(1,-1)
	1	(1,-1)	(-1,1)

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Nash Equilibrium as a Fixed Point

- Individual Best Response

$$\hat{B}_i(a) = \{b_i \in A_i : u_i(b_i, a_{-i}) \geq u_i(a_i, a_{-i}) \forall a_i \in A_i\}$$

- Synchronous Best Response

$$\hat{B}(a) = \underset{i \in N}{\times} \hat{B}_i(a)$$

- Nash Equilibrium as a fixed point

$$a^* = \hat{B}(a^*)$$

- Fixed point theorems can be used to establish existence of NE (see dissertation)
- NE can be solved by implied system of equations

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Best Response Analysis

Goal
$$u_i(c) = \left(B - \sum_{k \in N} c_k \right) c_i - K c_i$$

Best Response
$$c_i = \hat{B}_i(c) = \left(B - K - \sum_{k \in N \setminus i} c_k \right) / 2$$

Simultaneous System of Equations

c_1	+	$0.5\,c_2$	+	$0.5\,c_3$	+	$0.5\,c_4$	+	$0.5\,c_5$	=	$(B\text{-}K)/2$
$0.5\,c_1$	+	c_2	+	$0.5\,c_3$	+	$0.5\,c_4$	+	$0.5\,c_5$	=	$(B\text{-}K)/2$
$0.5\,c_1$	+	$0.5\,c_2$	+	c_3	+	$0.5\,c_4$	+	$0.5\,c_5$	=	$(B\text{-}K)/2$
$0.5\,c_1$	+	$0.5\,c_2$	+	$0.5\,c_3$	+	c_4	+	$0.5\,c_5$	=	$(B\text{-}K)/2$
$0.5\,c_1$	+	$0.5\,c_2$	+	$0.5\,c_3$	+	$0.5\,c_4$	+	c_5	=	$(B\text{-}K)/2$

Solution
$$\hat{c}_i = (B - K)/6 \;\forall i \in N$$

Generalization

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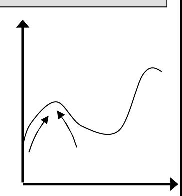
Significance of NE for CRNs

Theorem 4.1: NE and Cognitive Radio Network Steady States (*)
Given cognitive radio network $\langle N, A, \{u_i\}, \{d_i\}, T \rangle$ where all players are autonomously rational, if the game $\langle N, A, \{u_i\} \rangle$ has an NE a^*, then a^* is a fixed point for d.

Proof: Suppose a^* is not a fixed point. Then for some $i \in N$, there must be some $b_i \in d_i(a^*)$ with $b_i \neq a_i^*$ such that $u_i(b_i, a_{-i}^*) > u_i(a_i^*, a_{-i}^*)$. But this contradicts the assumption that a^* is an NE. Therefore, a^* must be a fixed point for d.

Autonomously Rational Decision Rule

$$b_i \in d_i(a),\; b_i \neq a_i \;\Rightarrow\; u_i(b_i, a_{-i}) > u_i(a_i, a_{-i})$$



- Why not "if and only if"?
  - Consider a self-motivated game with a local maximum and a hill-climbing algorithm.
  - For many decision rules, NE do capture all fixed points (see dissertation)
- Identifies steady-states for all "intelligent" decision rules with the same goal.
- Implies a mechanism for policy design while accommodating differing implementations
  - Verify goals result in desired performance
  - Verify radios act intelligently

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Optimality

- In general we assume the existence of some design objective function J:A→ℝ
- The desirableness of a network state, *a*, is the value of *J*(a).
- In general maximizers of *J* are unrelated to fixed points of *d*.

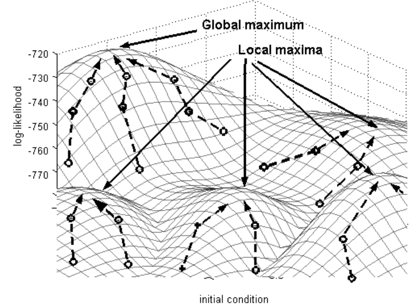

Figure from Fig 2.6 in I. Akbar, "Statistical Analysis of Wireless Systems Using Markov Models," PhD Dissertation, Virginia Tech, January 2007

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Example Functions

- Utilitarian
  – Sum of all players' utilities
  – Product of all players' utilities
- Practical
  – Total system throughput
  – Average SINR
  – Maximum End-to-End Latency
  – Minimal sum system interference
- Objective can be unrelated to utilities

Utilitarian Maximizers

Γ	N	W
n	(9.6,9.6)	(3.2,21)
w	(21,3.2)	(7,7)

System Throughput Maximizers

Γ	N	W
n	(9.6,9.6)	(3.2,21)
w	(21,3.2)	(7,7)

Interference Minimization

Γ	N	W
n	(9.6,9.6)	(3.2,21)
w	(21,3.2)	(7,7)

Price of Anarchy (Factor)

$$Ⓐ = \frac{\text{Performance of Centralized Algorithm Solution}}{\text{Performance of Distributed Algorithm Solution}}$$

$$Ⓐ \geq \mathbf{1}$$

- Centralized solution always at least as good as distributed solution
  - Like ASIC is always at least as good as DSP
- Ignores costs of implementing algorithms
  - Sometimes centralized is infeasible (e.g., routing the Internet)
  - Distributed can sometimes (but not generally) be more costly than centralized

Γ	N	W
n	(9.6,9.6)	(3.2,21)
w	(21,3.2)	(7,7)

$$Ⓐ = \frac{9.6}{7}$$

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Implications

- Best of All Possible Worlds
  - Low complexity distributed algorithms with low anarchy factors
- Reality implies mix of methods
  - Hodgepodge of mixed solutions
    - Policy – bounds the price of anarchy
    - Utility adjustments – align distributed solution with centralized solution
    - Market methods – sometimes distributed, sometimes centralized
    - Punishment – sometimes centralized, sometimes distributed, sometimes both
    - Radio environment maps –"centralized" information for distributed decision processes
  - Fully distributed
    - Potential game design – really, the panglossian solution, but only applies to particular problems

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Pareto efficiency (optimality)

- **Formal definition**: An action vector a^* is ***Pareto efficient*** if there exists no other action vector *a*, such that every radio's valuation of the network is at least as good and at least one radio assigns a higher valuation
- **Informal definition:** An action tuple is *Pareto efficient* if some radios must be hurt in order to improve the payoff of other radios.
- **Important note**
  - Like design objective function, unrelated to fixed points (NE)
  - Generally less specific than evaluating design objective function

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Example Games

Legend: Pareto Efficient

NE

NE + PE

	a_2	b_2
a_1	1,1	-5,5
b_1	5,-5	-1,-1

	a_2	b_2
a_1	1,1	-5,5
b_1	5,-5	3, 3

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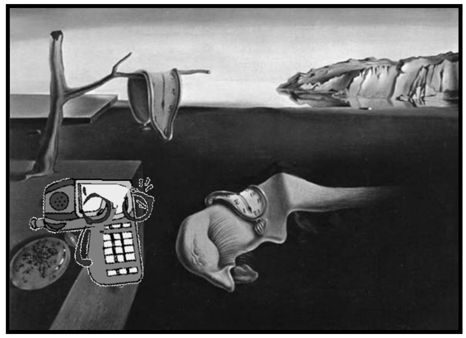
The Notion of Time and Imperfections in Games and Networks

Extensive Form Games, Repeated Games, Convergence Concepts in Normal Form Games, Trembling Hand Games, Noisy Observations



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Model Timing Review

- When decisions are made also matters and different radios will likely make decisions at different time
- T_j – when radio j makes its adaptations
  - Generally assumed to be an infinite set
  - Assumed to occur at discrete time
    - Consistent with DSP implementation
- $T = T_1 \cup T_2 \cup \cdots \cup T_n$
- $t \in T$

Decision timing classes

- Synchronous
  - All at once
- Round-robin
  - One at a time in order
  - Used in a lot of analysis
- Random
  - One at a time in no order
- Asynchronous
  - Random subset at a time
  - Least overhead for a network

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Repeated Games

- Same game is *repeated*
  - Indefinitely
  - Finitely
- Players consider discounted payoffs across multiple stages
  - Stage *k*

    $\tilde{u}_i\left(a^k\right) = \delta^k u_i\left(a^k\right)$
  - Expected value over all future stages

    $\hat{u}_i\left(\left(a^k\right)\right) = \sum_{k=0}^{\infty} \delta^k u_i\left(a^k\right)$

Stage 1

Γ	*N*	*W*
n	(9.6,9.6)	(3.2,21)
w	(21,3.2)	(7,7)

Stage 2

Γ	*N*	*W*
n	(9.6,9.6)	(3.2,21)
w	(21,3.2)	(7,7)

Stage *k*

Γ	*N*	*W*
n	(9.6,9.6)	(3.2,21)
w	(21,3.2)	(7,7)

Myopic Processes

- Players have no knowledge about utility functions, or expectations about future play, typically can observe or infer current actions
- Best response dynamic – maximize individual performance presuming other players' actions are fixed
- Better response dynamic – improve individual performance presuming other players' actions are fixed
- Interesting convergence results can be established

Paths and Convergence

- *Path* [Monderer_96]
  - A path in Γ is a sequence $\gamma = (a^0, a^1, \ldots)$ such that for every $k \geq 1$ there exists a unique player such that the strategy combinations (a^{k-1}, a^k) differs in exactly one coordinate.
  - Equivalently, a path is a sequence of unilateral deviations. When discussing paths, we make use of the following conventions.
  - Each element of γ is called a *step*.
  - a^0 is referred to as the *initial* or *starting point* of γ.
  - Assuming γ is finite with *m* steps, a^m is called the *terminal point* or *ending point* of γ and say that γ has *length m*.
- *Cycle* [Voorneveld_96]
  - A finite path $\gamma = (a^0, a^1, \ldots, a^k)$ where $a^k = a^0$

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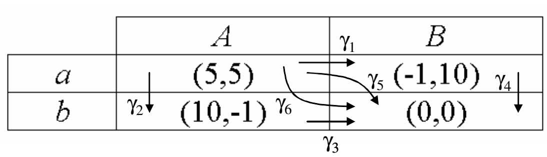
Improvement Paths

- Improvement Path
  - A path $\gamma = (a^0, a^1, \ldots)$ where for all $k \geq 1$, $u_i(a^k) > u_i(a^{k-1})$ where *i* is the unique deviator at *k*
- Improvement Cycle
  - An improvement path that is also a cycle
  - See the DFS example



$\gamma_1 = ((a, A), (a, B))$	$\gamma_3 = ((b, A), (b, B))$	$\gamma_5 = (\gamma_1, (b, B))$
$\gamma_2 = ((a, A), (b, A))$	$\gamma_4 = ((a, B), (b, B))$	$\gamma_6 = (\gamma_1, (b, B))$

Convergence Properties

- Finite Improvement Property (FIP)
  - All improvement paths in a game are finite
- Weak Finite Improvement Property (weak FIP)
  - From every action tuple, there exists an improvement path that terminates in an NE.
- FIP implies weak FIP
- FIP implies lack of improvement cycles
- Weak FIP implies existence of an NE

Examples

Game with FIP

	A	B
a	1,-1	0,2
b	-1,1	2,2

Weak FIP but not FIP

	A	B	C
a	1,-1	-1,1	0,2
b	-1,1	1,-1	1,2
c	2,0	2,1	2,2

Implications of FIP and weak FIP

- Assumes radios are incapable of reasoning ahead and must react to internal states and current observations
- Unless the game model of a CRN has weak FIP, then no autonomously rational decision rule can be guaranteed to converge from all initial states under random and round-robin timing (Theorem 4.10 in dissertation).
- If the game model of a CRN has FIP, then ALL autonomously rational decision rules are guaranteed to converge from all initial states under random and round-robin timing.
  - And asynchronous timings, but not immediate from definition
- More insights possible by considering more refined classes of decision rules and timings

Decision Rules

Definition 4.10: *Best Response Dynamic*
A decision rule $d_i : A \rightarrow A_i$ is a best response dynamic if each adaptation would maximize the radio's utility if all other radios continued to implement the same waveforms, i.e., $d_i(a) \in \{b_i \in A_i : u_i(b_i, a_{-i}) \geq u_i(a_i, a_{-i}) \forall a_i \in A_i\}$

Definition 4.11: *Better Response Dynamic*
A decision rule $d_i : A \rightarrow A_i$ is a better response dynamic if each adaptation would improve the radio's utility if all other radios continued to implement the same waveforms, i.e., $d_i(a) \in \{b_i \in A_i : u_i(b_i, a_{-i}) > u_i(a_i, a_{-i})\}$.

Definition 4.13: *Friedman's Random Better Response* [Friedman_01]
Player *i* chooses an action from $A_i \backslash b_i$ where b_i is player *i*'s current action according to a uniform random distribution. If the chosen action would improve the utility of player *i*, it is implemented, otherwise, the player continues to play b_i.

Definition 4.12: *Random Better Response Dynamic* (*)
A decision rule $d_i : A \rightarrow A_i$ is a random better response dynamic if for each $t_i \in T_i$, radio *i* chooses an action from A_i where each action has a nonzero probability of being chosen and implements the action if it would improve its utility.

Absorbing Markov Chains and Improvement Paths

- Sources of randomness
  - Timing (Random, Asynchronous)
  - Decision rule (random decision rule)
  - Corrupted observations
- An NE is an absorbing state for autonomously rational decision rules.
- Weak FIP implies that the game is an absorbing Markov chain as long as the NE terminating improvement path always has a nonzero probability of being implemented.
- This then allows us to characterize
  - convergence rate,
  - probability of ending up in a particular NE,
  - expected number of times a particular transient state will be visited

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Connecting Markov models, improvement paths, and decision rules

- Suppose we need the path $\gamma = (a^0, a^1, \ldots a^m)$ for convergence by weak FIP.
- Must get right sequence of players and right sequence of adaptations.
- Friedman Random Better Response
  - Random or Asynchronous
    - Every sequence of players have a chance to occur
    - Random decision rule means that all improvements have a chance to be chosen
  - Synchronous not guaranteed

Γ	A	B
a	(1,1)	(0,0)
b	(0,0)	(1,1)

- Alternate random better response (chance of choosing same action)
  - Because of chance to choose same action, every sequence of players can result from every decision timing.
  - Because of random choice, every improvement path has a chance of occurring

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Convergence Results (Finite Games)

	Timings			
Decision Rules	Round-Robin	Random	Synchronous	Asynchronous
Best Response	1,3	1,3	1	1,3
Exhaustive Better Response	3	3	-	3
Random Better Response[(a)]	1,2,3	1,2,3	1,2,3	1,2,3
Random Better Response[(b)]	1,3	1,2,3	1	1,2,3

(a) Definition 4.12, (b) Definition 4.13, 1. IESDS, 2. Weak FIP, 3. FIP

- If a decision rule converges under round-robin, random, or synchronous timing, then it also converges under asynchronous timing.
- Random better responses converge for the most decision timings and the most surveyed game conditions.
  - Implies that non-deterministic procedural cognitive radio implementations are a good approach if you don't know much about the network.

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Trembling Hand ("Noise" in Games)

- Assumes players have a nonzero chance of making an error implementing their action.
  - Who has not accidentally handed over the wrong amount of cash at a restaurant?
  - Who has not accidentally written a "tpyo"?
- Related to errors in observation as erroneous observations cause errors in implementation (from an outside observer's perspective).

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Noisy decision rules

- Noisy utility $\tilde{u}_i(a,t) = u_i(a) + n_i(a,t)$

Trembling Hand

Definition 4.20: *Friedman's Noisy Random Better Response* [Friedman_01]
Player i chooses an action $a_i \in A_i \backslash b_i$ where b_i is player i's current action according to a uniform random distribution. If $u_i(a_i,a_{-i}) > u_i(b_i,a_{-i})$, then a_i is implemented, however, if $u_i(a_i,a_{-i}) \leq u_i(b_i,a_{-i})$, then player i still switches to a_i with nonzero probability ρ.

Observation Errors

Definition 4.21: *Noisy Best Response Dynamic* (*)
A decision rule $\tilde{d}_i : A \times T \to A_i$ is a noisy best response dynamic if each adaptation would maximize the radio's noisy utility if all other radios continued to implement the same waveforms, i.e., $\tilde{d}_i(a) \in \{b_i \in A_i : \tilde{u}_i(b_i, a_{-i}, t) \geq \tilde{u}_i(a_i, a_{-i}, t) \forall a_i \in A_i\}$

Definition 4.22: *Noisy Better Response Dynamic* (*)
A decision rule $\tilde{d}_i : A \times T \to A_i$ is a noisy better response dynamic if each adaptation would improve the radio's utility if all other radios continued to implement the same waveforms, i.e., $\tilde{d}_i(a) \in \{b_i \in A_i : \tilde{u}_i(b_i, a_{-i}, t) > \tilde{u}_i(a_i, a_{-i}, t)\}$.

Definition 4.23: *Noisy Random Better Response Dynamic* (*)
A decision rule $\tilde{d}_i : A \times T \to A_i$ is a random better response dynamic if for each $t_i \in T_i$, radio i chooses an action from A_i with nonzero probability and implements the action if it would improve $\tilde{u}_i$.

Implications of noise

- For random timing, [Friedman] shows game with noisy random better response is an ergodic Markov chain.
- Likewise other observation based noisy decision rules are ergodic Markov chains
  – Unbounded noise implies chance of adapting (or not adapting) to any action
  – If coupled with random, synchronous, or asynchronous timings, then CRNs with corrupted observation can be modeled as ergodic Makov chains.
  – Not so for round-robin (violates aperiodicity)
- Somewhat disappointing
  – No real steady-state (though unique limiting stationary distribution)

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DFS Example with three access points

- 3 access nodes, 3 channels, attempting to operate in band with least spectral energy.
- Constant power
- Link gain matrix

g_{ik}	1	2	3
1	1	0.5	0.1
2	0.5	1	0.3
3	0.1	0.3	1

- Noiseless observations

(f_1,f_1,f_1)	(f_1,f_1,f_2)	(f_1,f_2,f_1)	(f_1,f_2,f_2)	(f_2,f_1,f_1)	(f_2,f_1,f_2)	(f_2,f_2,f_1)	(f_2,f_2,f_2)	(f_2,f_1,f_1)
(0.6,0.8,0.4)	(0.5,0.5,0.0)	(0.1,0.0,0.1)	(0.0,0.3,0.3)	(0.0,0.3,0.3)	(0.1,0.0,0.1)	(0.5,0.5,0.0)	(0.6,0.8,0.4)	(0.0,0.3,0.3)

- Random timing

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Trembling Hand

- Transition Matrix, p=0.1

P	(f_1,f_1,f_1)	(f_1,f_1,f_2)	(f_1,f_2,f_1)	(f_1,f_2,f_2)	(f_2,f_1,f_1)	(f_2,f_1,f_2)	(f_2,f_2,f_1)	(f_2,f_2,f_2)
(f_1,f_1,f_1)	0	1/3	1/3	0	1/3	0	0	0
(f_1,f_1,f_2)	1/30	3/10	0	1/3	0	1/3	0	0
(f_1,f_2,f_1)	1/30	0	9/10	1/30	0	0	1/30	0
(f_1,f_2,f_2)	0	1/30	1/3	3/5	0	0	0	1/30
(f_2,f_1,f_1)	1/30	0	0	0	3/5	1/3	1/30	0
(f_2,f_1,f_2)	0	1/30	0	0	1/30	9/10	0	1/30
(f_2,f_2,f_1)	0	0	1/3	0	1/3	0	3/10	1/30
(f_2,f_2,f_2)	0	0	0	1/3	0	1/3	1/3	0

- Limiting distribution

(f_1,f_1,f_1)	(f_1,f_1,f_2)	(f_1,f_2,f_1)	(f_1,f_2,f_2)	(f_2,f_1,f_1)	(f_2,f_1,f_2)	(f_2,f_2,f_1)	(f_2,f_2,f_2)
0.0161	0.0293	0.3846	0.0699	0.0699	0.3846	0.0293	0.0161

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Noisy Best Response

- Transition Matrix, $\mathcal{N}(0,1)$ Gaussian Noise

P	(f_1,f_1,f_1)	(f_1,f_1,f_2)	(f_1,f_2,f_1)	(f_1,f_2,f_2)	(f_2,f_1,f_1)	(f_2,f_1,f_2)	(f_2,f_2,f_1)	(f_2,f_2,f_2)
(f_1,f_1,f_1)	0.3367	0.2038	0.2381	0	0.2214	0	0	0
(f_1,f_1,f_2)	0.1295	0.4813	0	0.1854	0	0.2038	0	0
(f_1,f_2,f_1)	0.0953	0	0.6273	0.1479	0	0	0.1295	0
(f_1,f_2,f_2)	0	0.1479	0.1854	0.5548	0	0	0	0.1119
(f_2,f_1,f_1)	0.1119	0	0	0	0.5548	0.1854	0.1479	0
(f_2,f_1,f_2)	0	0.1295	0	0	0.1479	0.6273	0	0.0953
(f_2,f_2,f_1)	0	0	0.2038	0	0.1854	0	0.4813	0.1295
(f_2,f_2,f_2)	0	0	0	0.2214	0	0.2381	0.2038	0.3367

- Limiting stationary distributions

	(f_1,f_1,f_1)	(f_1,f_1,f_2)	(f_1,f_2,f_1)	(f_1,f_2,f_2)	(f_2,f_1,f_1)	(f_2,f_1,f_2)	(f_2,f_2,f_1)	(f_2,f_2,f_2)
σ=1.00	0.0709	0.1120	0.1765	0.1406	0.1406	0.1765	0.1120	0.0709
σ=0.50	0.0540	0.1040	0.1984	0.1436	0.1436	0.1984	0.1040	0.0540
σ=0.10	0.0129	0.0647	0.2857	0.1366	0.1366	0.2857	0.0647	0.0129
σ=0.05	0.0033	0.0397	0.3387	0.1183	0.1183	0.3387	0.0397	0.0033
σ=0.01	0	0.002	0.46	0.038	0.038	0.46	0.002	0

Comment on Noise and Observations

- Cardinality of goals makes a difference for cognitive radios
  - Probability of making an error is a function of the difference in utilities
  - With ordinal preferences, utility functions are just useful fictions
    - Might as well assume a trembling hand
- Unboundedness of noise implies that no state can be absorbing for most decision rules
- NE retains significant predictive power
  - While CRN is an ergodic Markov chain, NE (and the adjacent states) remain most likely states to visit
  - Stronger prediction with less noise
  - Also stronger when network has a Lyapunov function
  - Exception - elusive equilibria ([Hicks_04])

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Items to Remember

Game	⇔	Cognitive radio network
Player	⇔	Cognitive radio
Actions	⇔	Actions
Utility function	⇔	Goal
Outcome space	⇔	Outside world
Utility function arguments	⇔	Observations/orientation
Order of play	⇔	Adaptation timings

- NE are always fixed points for self-interested adaptations
  - But may not be ALL fixed points
- Many ways to measure optimality
- Randomness helps convergence
- Unbounded noise implies network has a theoretically non-zero chance to visit every possible state

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